ANCHOR:

A Memory-Based Model of Direct Psychophysical Scaling

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I’m Glad to Meet You

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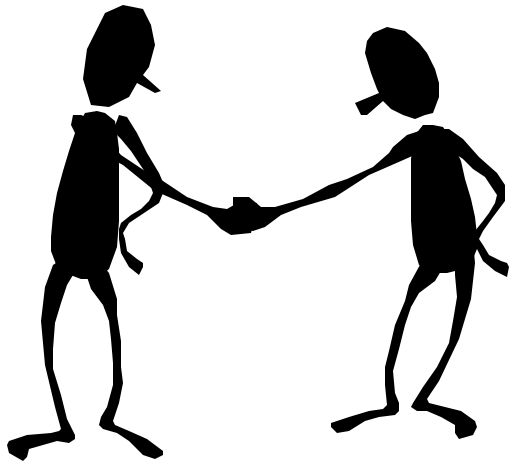

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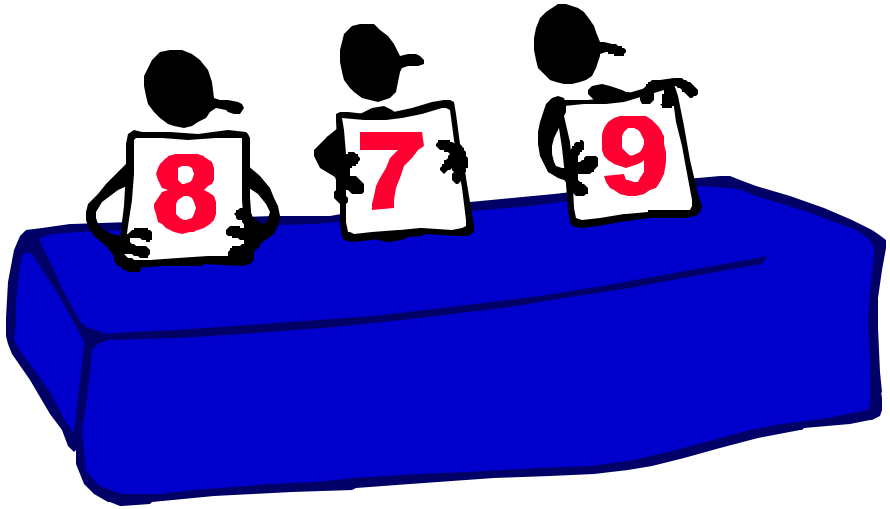
Plan of the Talk

- Empirical phenomena
- ANCHOR: theory and model
- Data fits
- Parameter estimation and individual differences

The Category-Rating Task

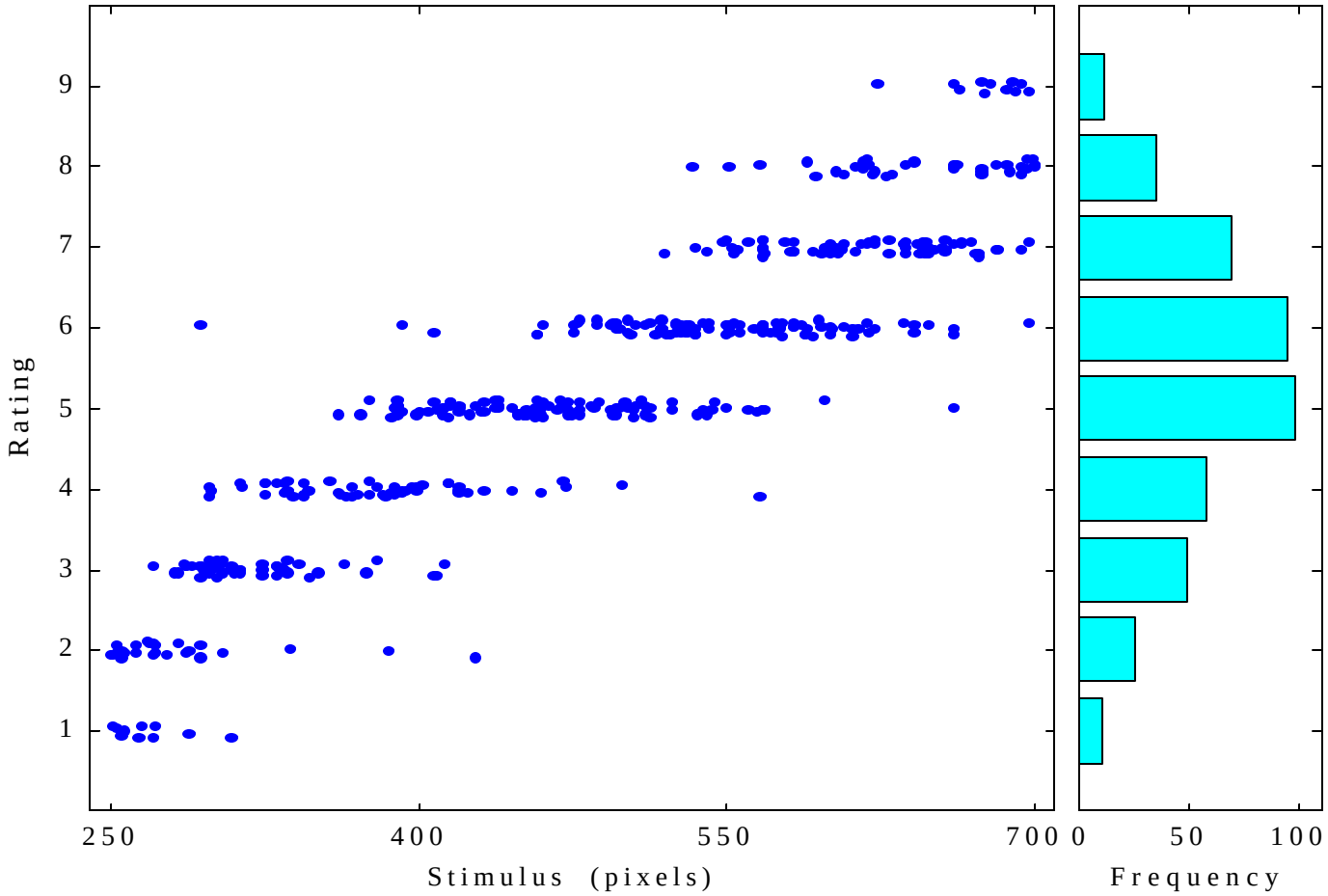
- “1”, “2”, …, “9”
- “very dissimilar”, …, “very similar”
- “strongly disagree”, …, “strongly agree”
- …



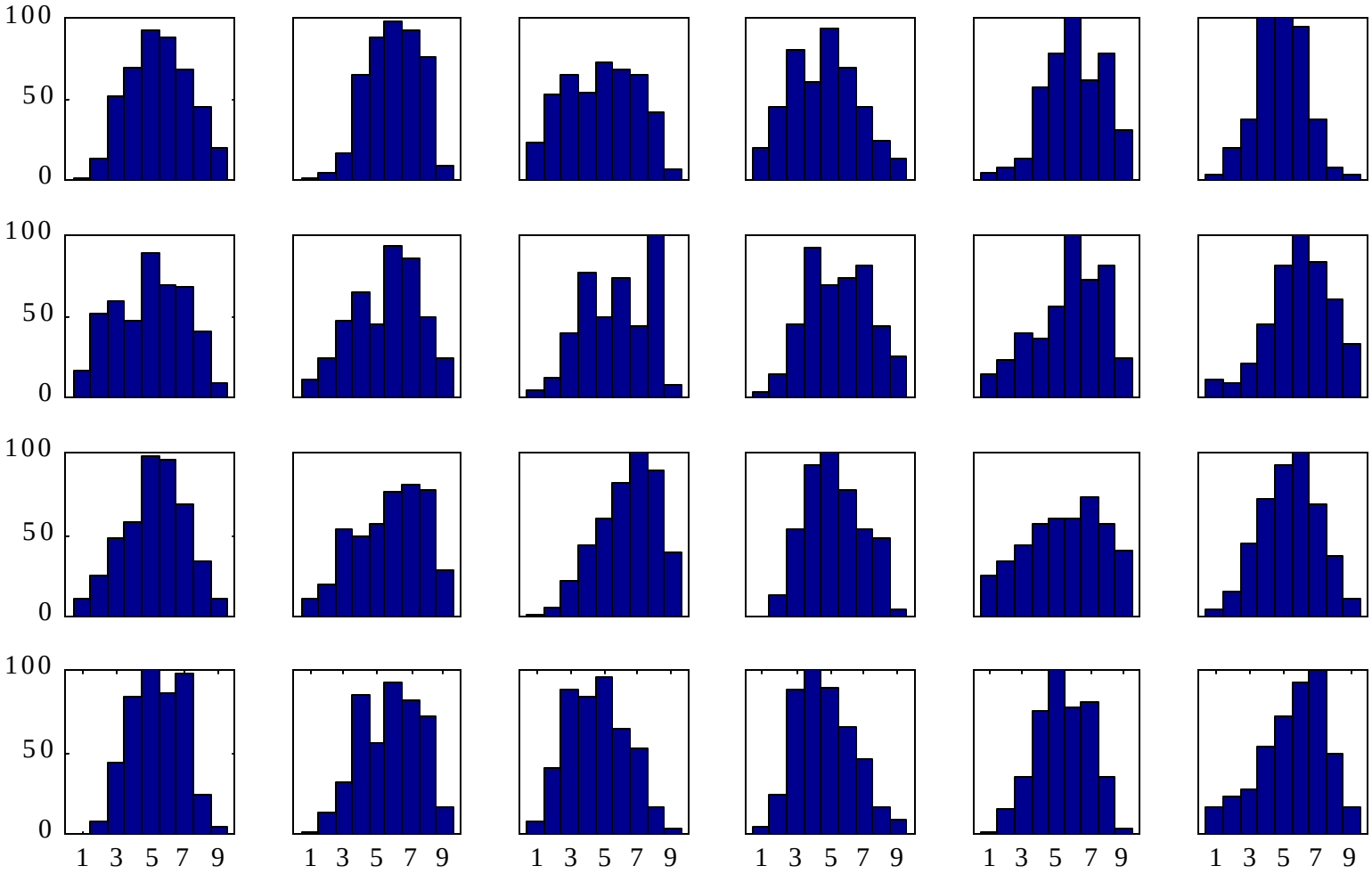
Empirical Phenomena

- Stevens' power law
- sequential effects
- context effects
- memory effects
- range effects
- edge effects
- …

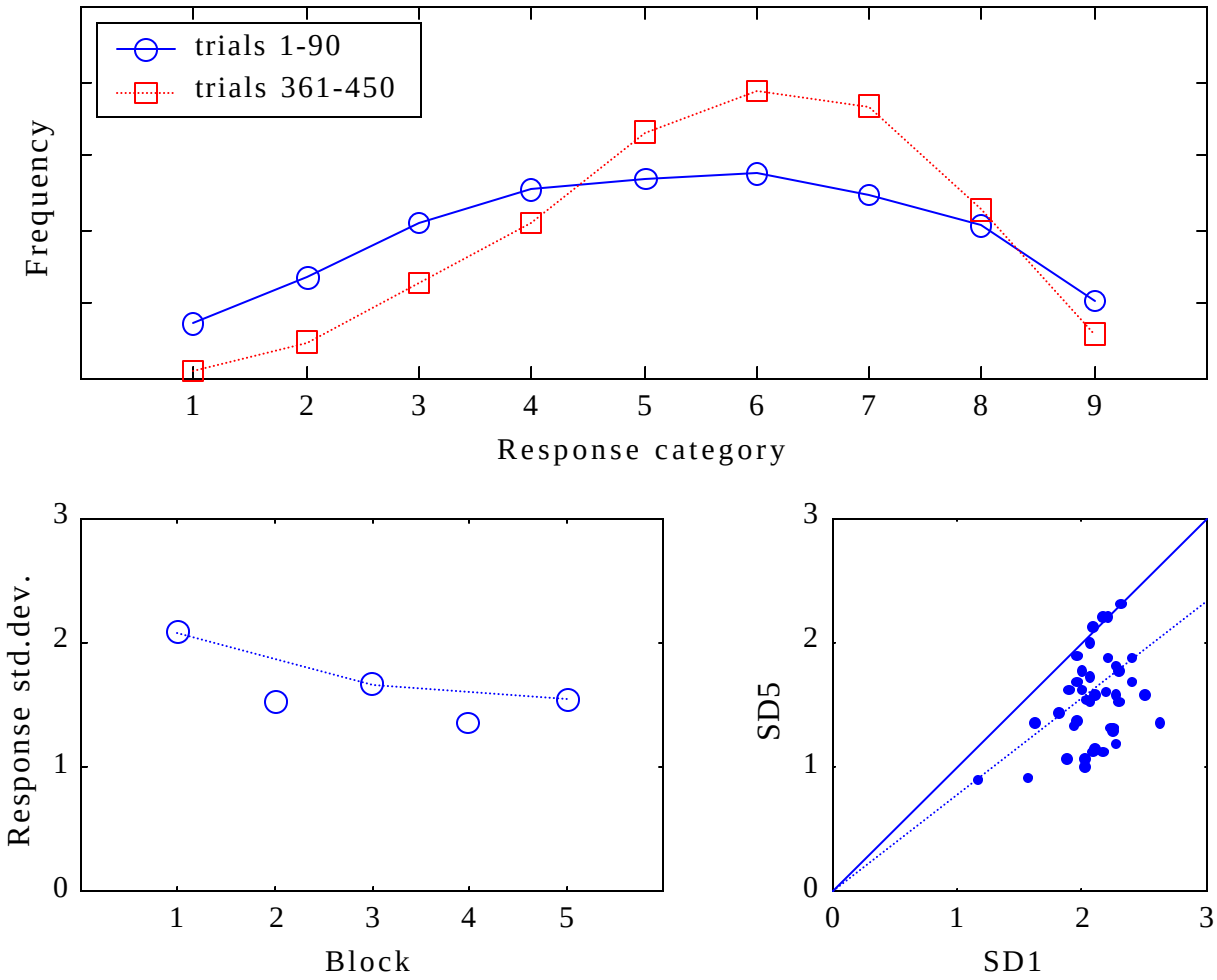
Typical Data Set



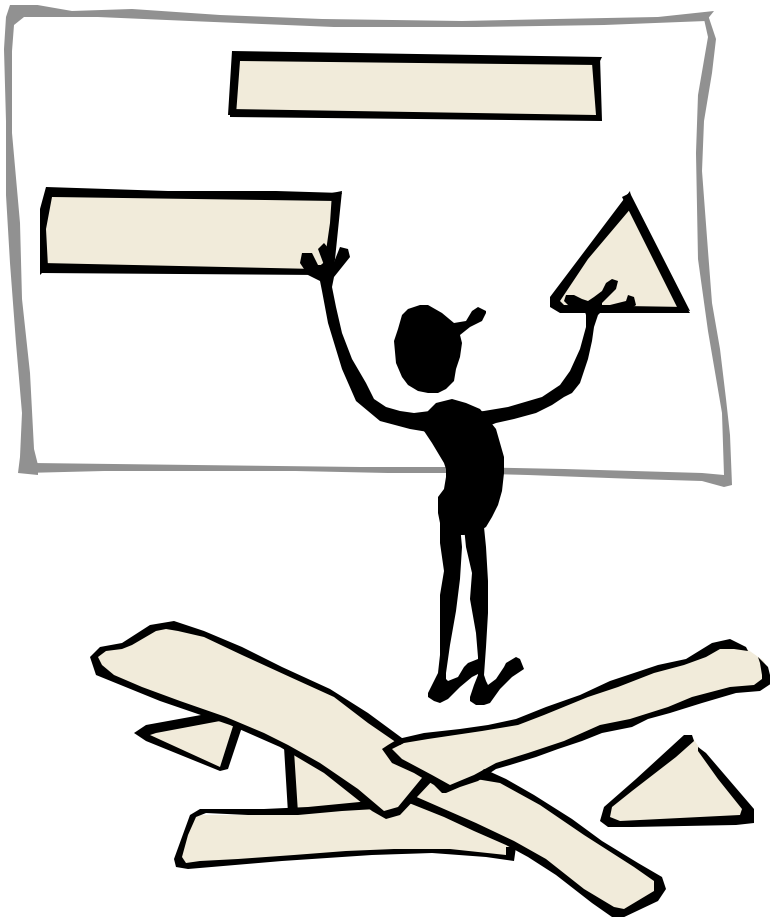
Response Distributions



Nonuniformity Increases with Time

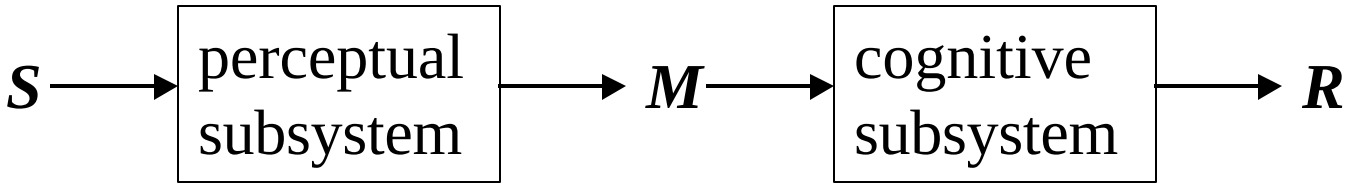


ANCHOR: Theory and Model

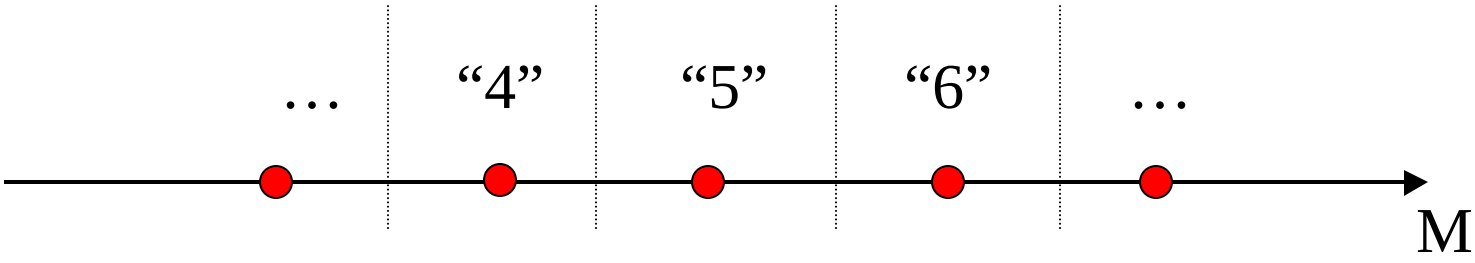


Two Main Steps

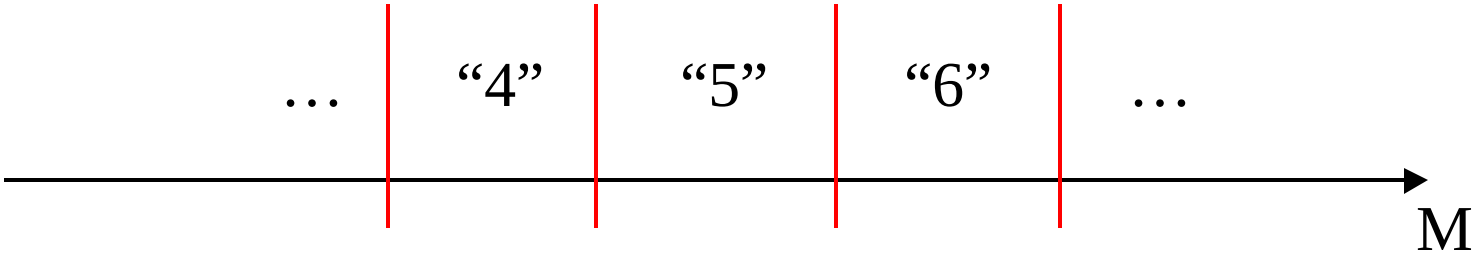
- perceive the stimulus
- report the subjective percept



Prototype-based Categories



Criterion-based Categories



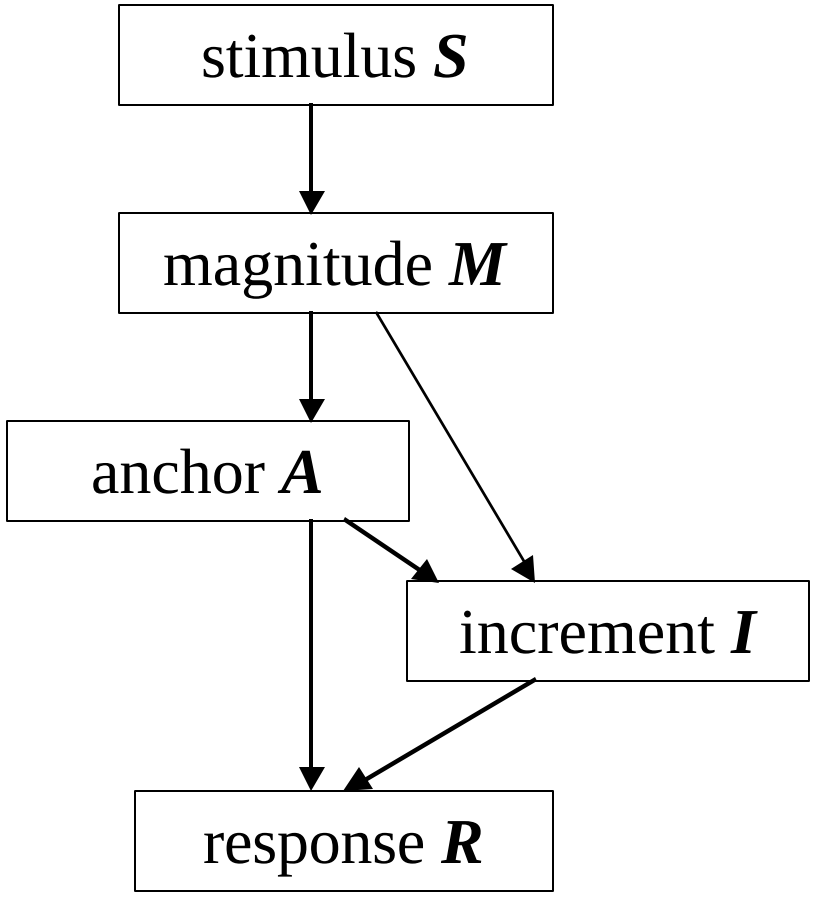
ANCHOR Principles

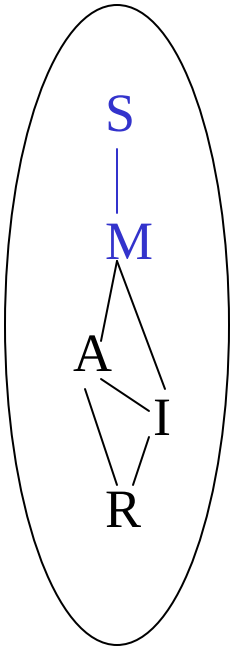
1. Internal continuum of *magnitudes*
2. Content-addressable memory
   anchor = <M,R> association
3. Explicit corrections
4. Obligatory incremental learning

Talk-aloud Protocol

I see the dots…
The distance looks like a 7…
No, it's too short for a 7; I'll give it a 6.

Dependencies among Variables



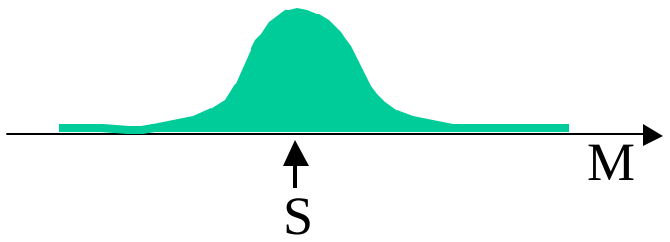


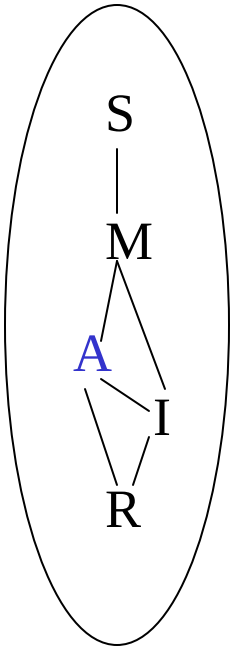
Perceptual Equation

$$M = aS\left(1 + k_p e_p\right)$$

multiplica-
tive noise

Each stimulus S defines a whole distribution of magnitudes.

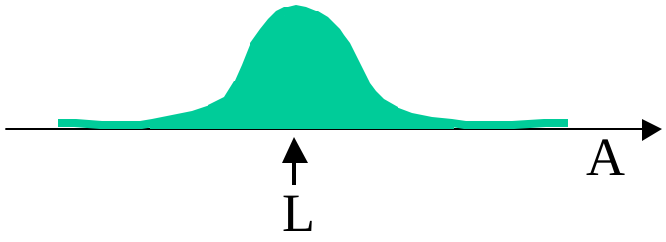


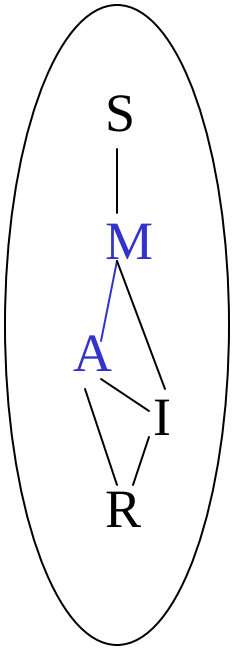


Anchor magnitudes are noisy too

$$A = L\left(1 + k_m e_m\right)$$

multiplica-tive noise





Anchor Selection

$$G_i = -\underbrace{\left|M - A_i\right|}_{\text{similarity}} + \underbrace{HB_i}_{\text{history}}$$

$$P_i = \frac{\exp(G_i / T)}{\sum_j \exp(G_j / T)}$$

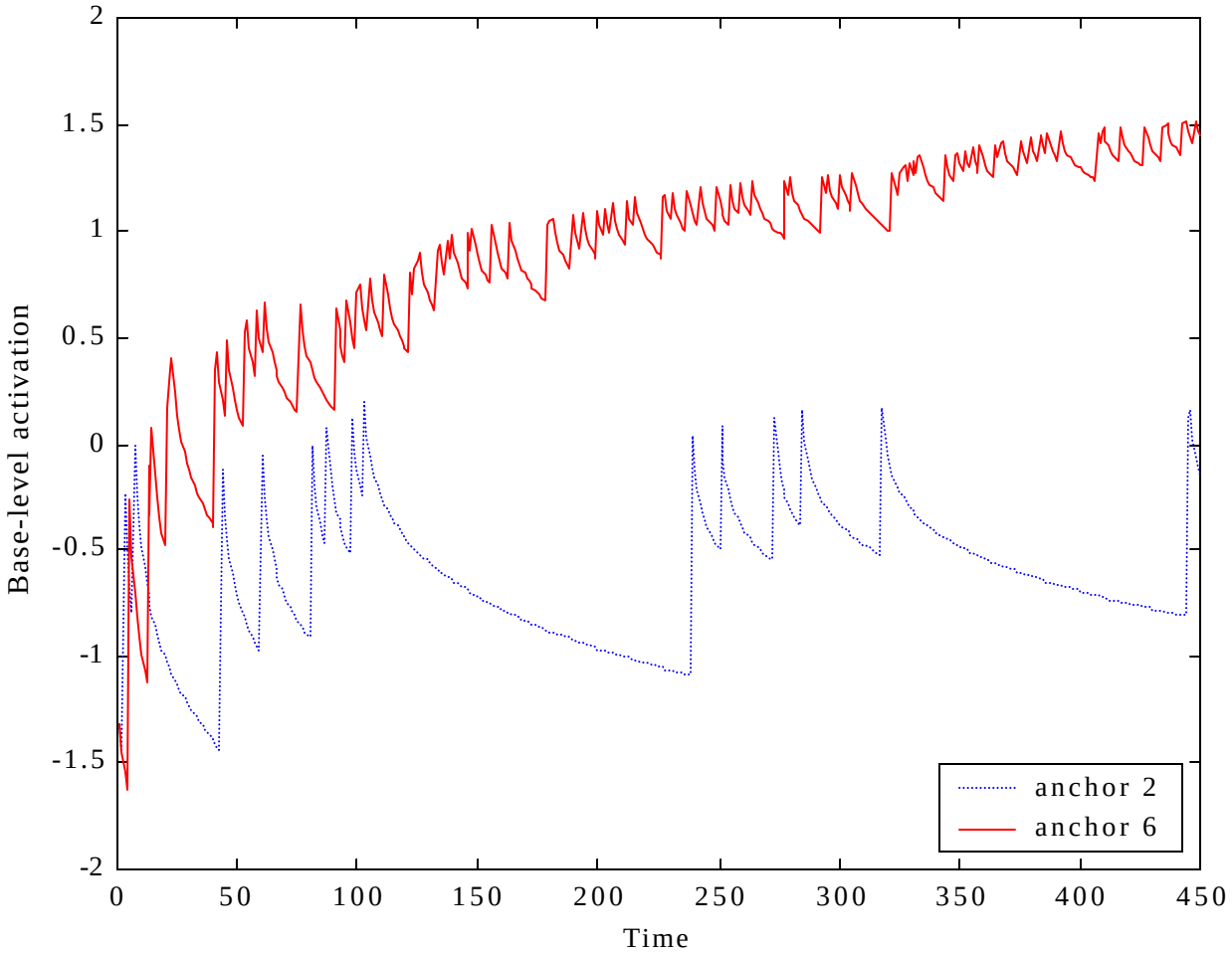
Anchor Selection Highlights

- *general memory* mechanism
- stochastic (*softmax* rule)
- depends on the *similarity* b/n the target and each of the anchors
- depends on the *availability*
  - recency
  - strength

$$G_i = -\left|M - A_i\right| + HB_i$$

$$P_i = \frac{\exp(G_i / T)}{\sum_j \exp(G_j / T)}$$

Dynamic Availability



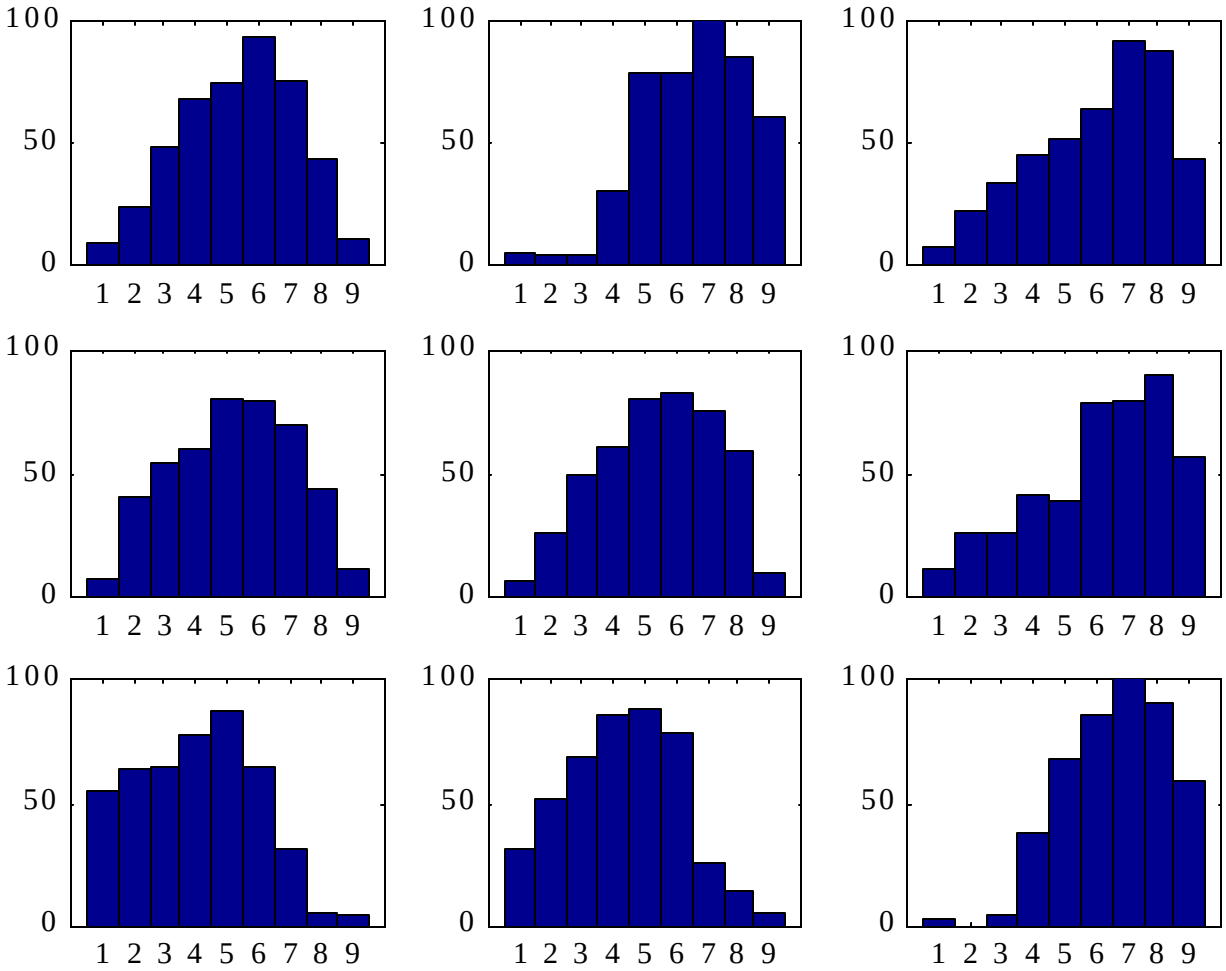
Base-Level Activation

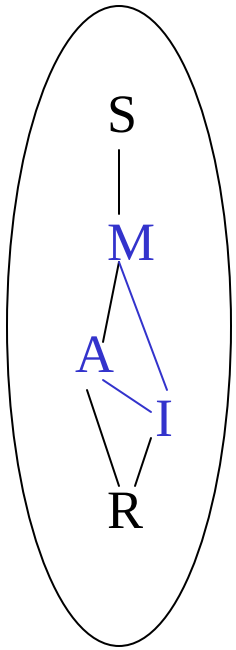
$$B = \ln\left[\sum_{l=1}^{n} t_l^{-d}\right]$$

$$B = \ln\left[t_{last}^{-d} + \frac{n.\left(t_{life}^{1-d} - t_{last}^{1-d}\right)}{(1-d).(t_{life} - t_{life})}\right]$$

$$B = \ln\left[t_{last}^{-0.5} + \frac{2n}{(\sqrt{t_{life}} + \sqrt{t_{last}})}\right]$$

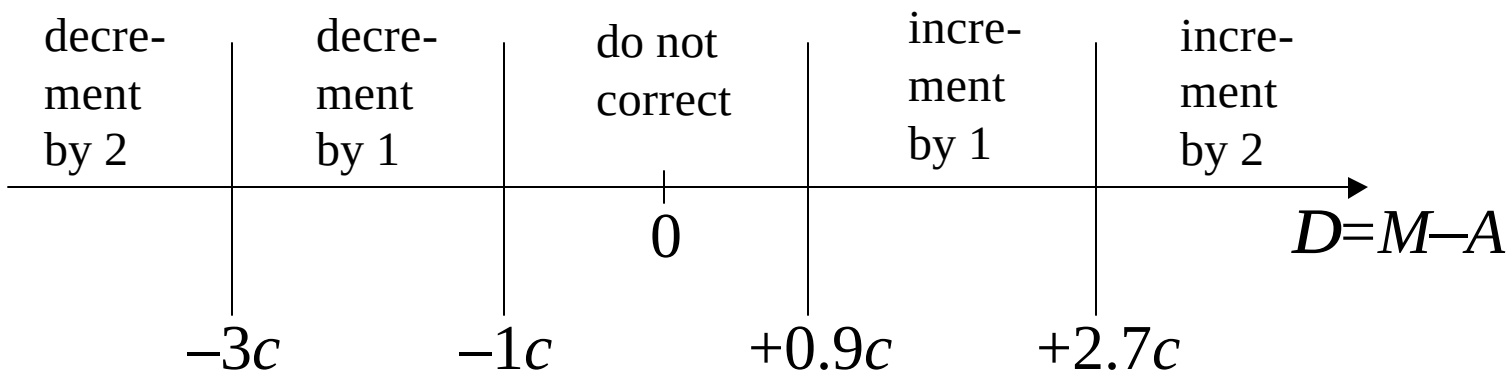
Category Strength (Model)





Correction Mechanism

- explicit strategy
- binds the anchors together
- redistribution of strength
- introduces prior knowledge

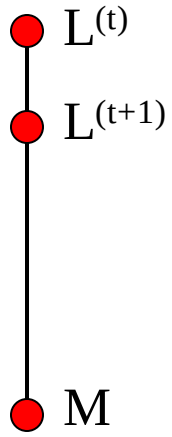


Question

The response has been produced. Is this the end of the trial?

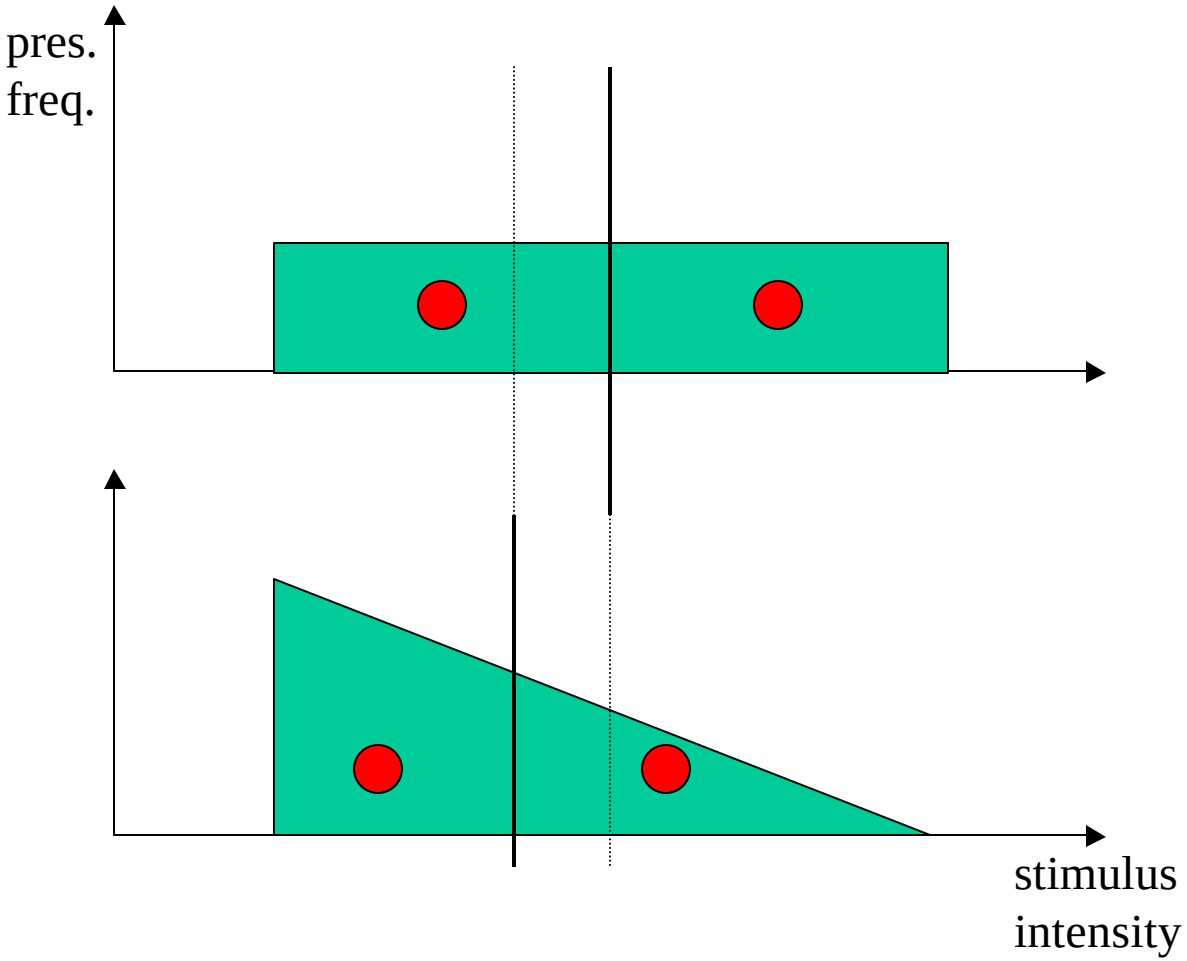
Updating the Anchor Locations

$$L^{(t+1)} = aM + (1-a)L^{(t)}$$

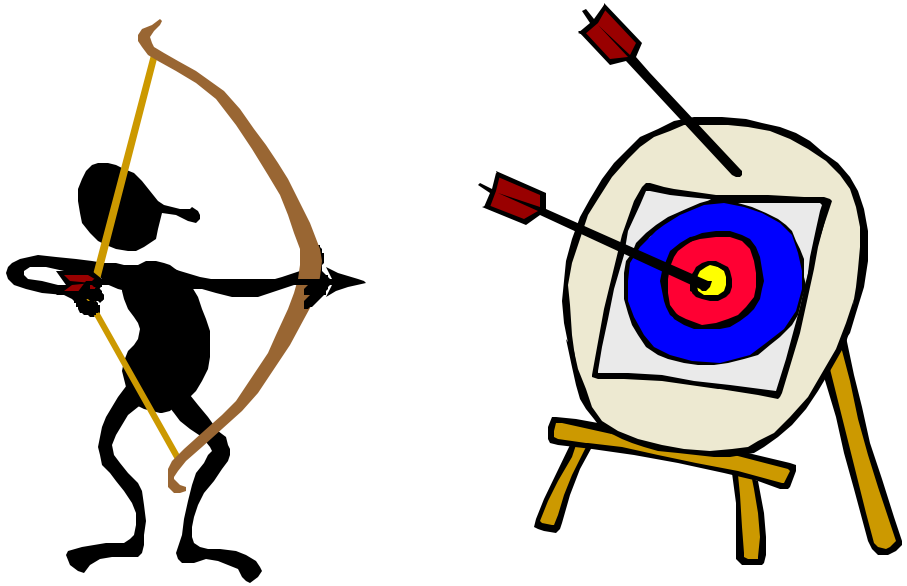


- a form of competitive learning
- anchors become *weighted prototypes*
- consistency of responses
- track the density → context effects

Context Effects



Testing the Model



Simulation Experiments

- the basic unit of analysis is the individual
- suite of statistical measures
- 40+24 Ss $\rightarrow$ 40+24 parameter sets
- 40+24 stimulus sequences; the same that were shown to the human participants
- (40+24) x 100 runs of the model

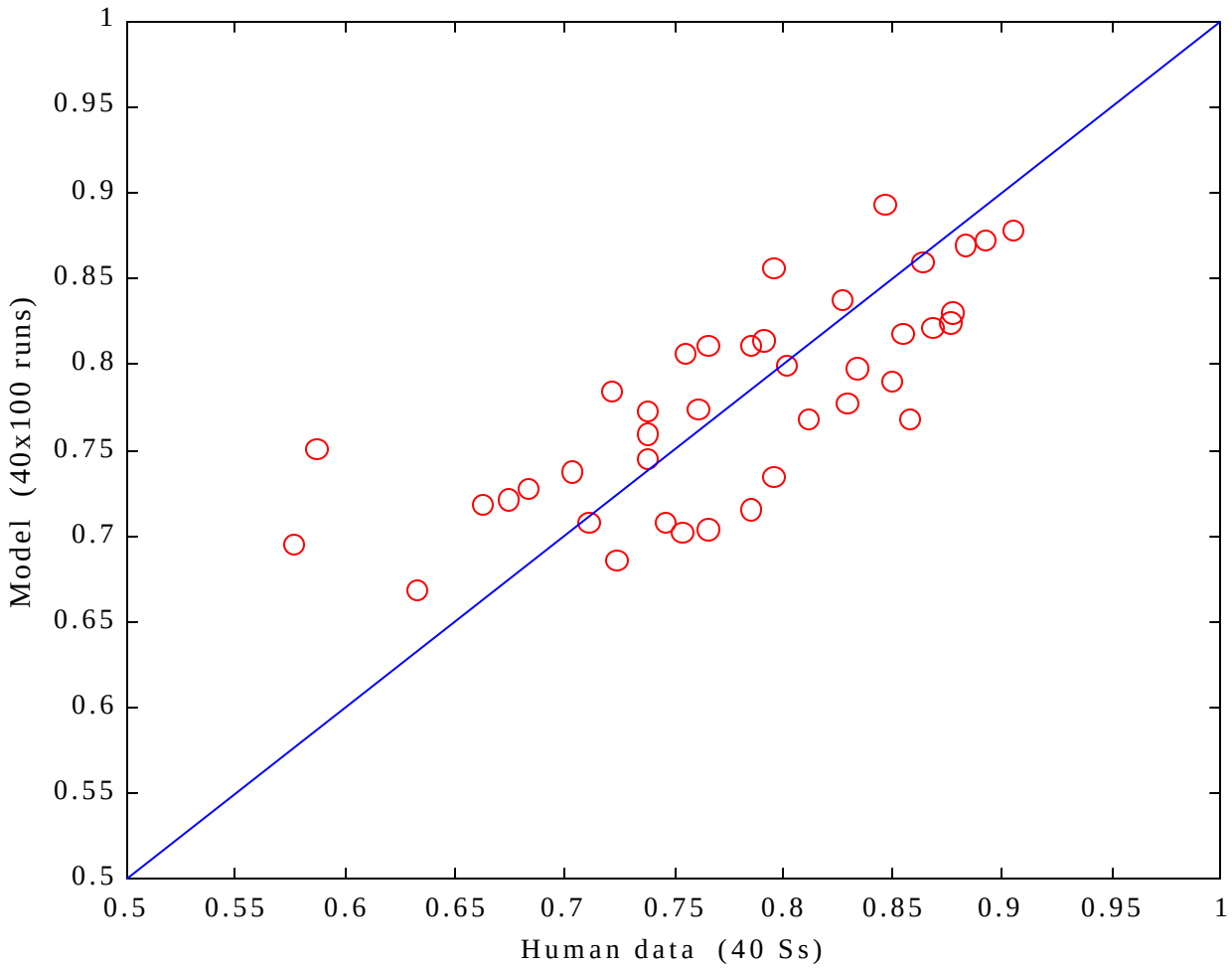
Model Fits: Category Rating

Statistic	**empir**	**model**	**R**
Accuracy (R^2)	0.78	0.78	.75
Mean response	5.56	5.30	–.16
Non-uniformity (s.d.)	1.77	1.85	.71
Nonstationary s.d.	0.55	0.29	.48
Sequential eff (acf_{resid})	0.34	0.15	.32
Context eff, Gr1 (Δ_{ARL})	–0.34	–0.47	–.17
Context eff, Gr2 (Δ_{ARL})	+0.04	+0.19	.34

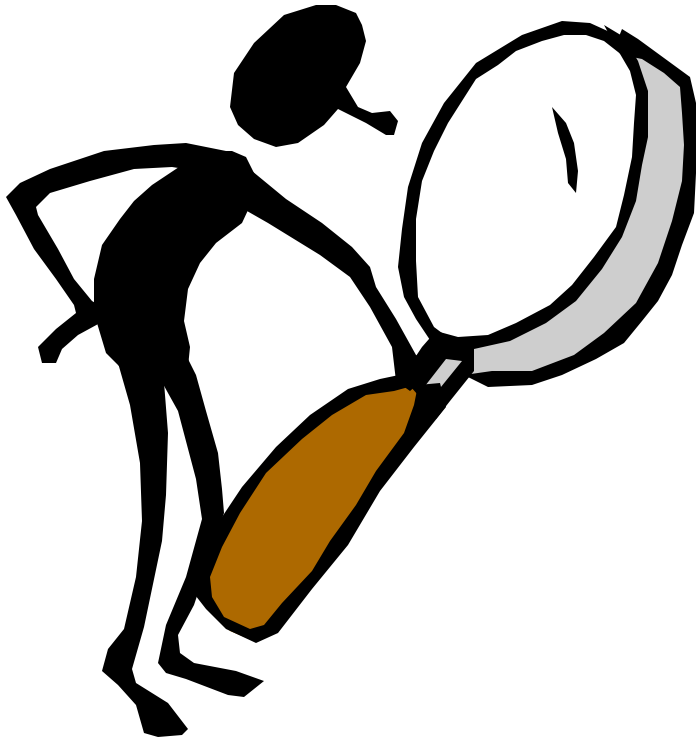
Model Fits: Absolute Identification

Statistic	**empir**	**model**	**R**
Accuracy (R^2)	0.90	0.86	.84
Transm. info (T)	1.69	1.44	.91
Mean response	5.03	5.03	–.18
Non-uniformity (s.d.)	2.40	2.49	.65
Nonstationary s.d.	0.01	–0.01	.12
Sequential eff (acf_{resid})	0.06	0.00	.03
Context eff, Gr1 (Δ_{ARL})	+0.19	+0.10	–.10
Context eff, Gr2 (Δ_{ARL})	–0.07	–0.11	.07

Accuracy (R^2)


Model (40x100 runs)
Human data (40 Ss)
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.5
0.55
0.6
0.65
0.7
0.75
0.8
0.85
0.9
0.95
1

Parameter Search

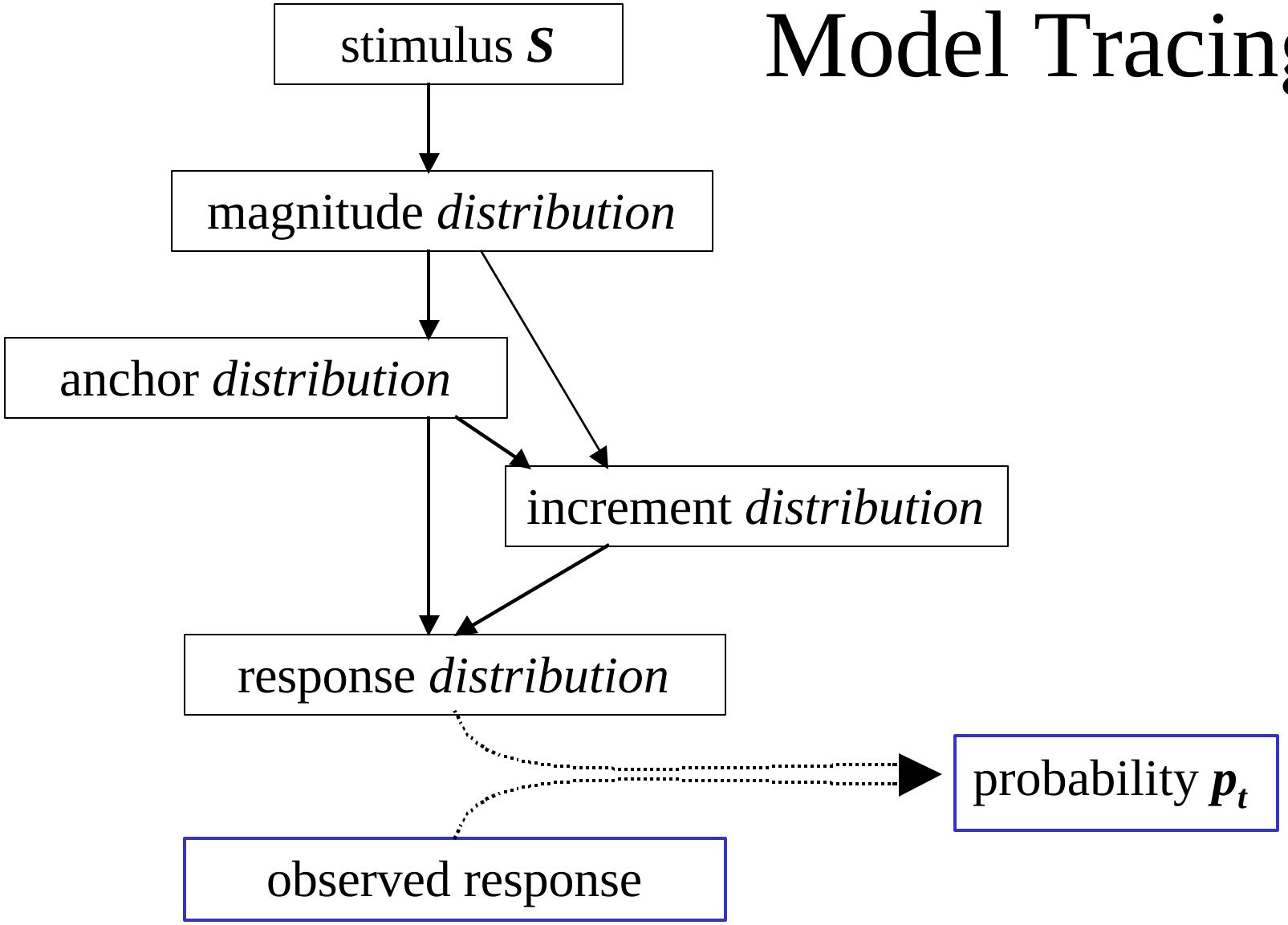


4+3 Parameters

- memory noise k_m
- softmax temperature T
- history weight H
- correction cutoff c

- perceptual noise k_p=0.04
- learning rate a=0.2
- correction bias c^+/c^-=0.9

Model Tracing



Maximize the Log-likelihood

$$L = -\sum_{t=1}^{450} \ln\left[p_t\right]$$

where $\boldsymbol{p_t}$ is the probability that the model produces on trial $\boldsymbol{t}$ the response that was produced by the human participant

Parameter Search

- Gradient descent on $\boldsymbol{L}$
- Individual parameter set optimized for each participant
- Reliability of the method tested on synthetic data
- Sensitivity analyses
- Discourages “fishing”

ANCHOR Principles

1. Internal continuum of *magnitudes*
2. Content-addressable memory
   anchor = <M,R> association
3. Explicit corrections
4. Obligatory incremental learning

The End

Highlights of the Talk

- Non-stationary processing
- Context effects in opposite directions
- Integration of psychophysics and memory
- Memory-based model
- Incremental learning algorithms

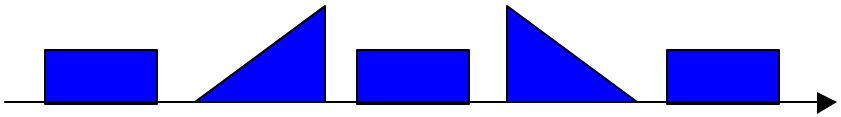
Related Tasks

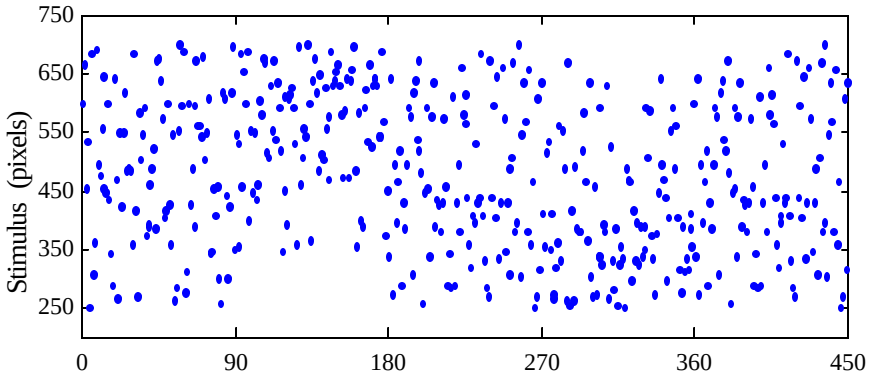
- **category rating**
- magnitude estimation
- absolute identification

- perceptual discrimination
- categorization
- pair-associate learning

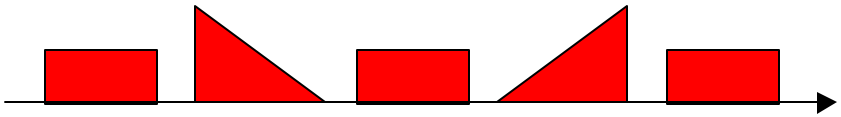
Experimental Design

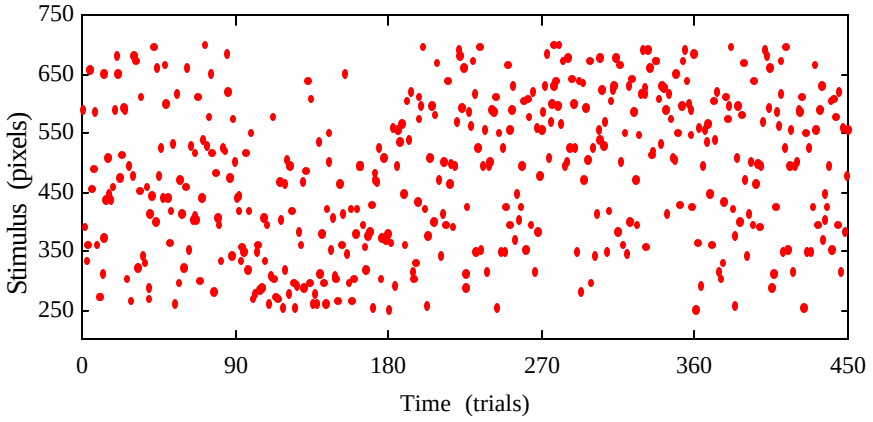
Group 1





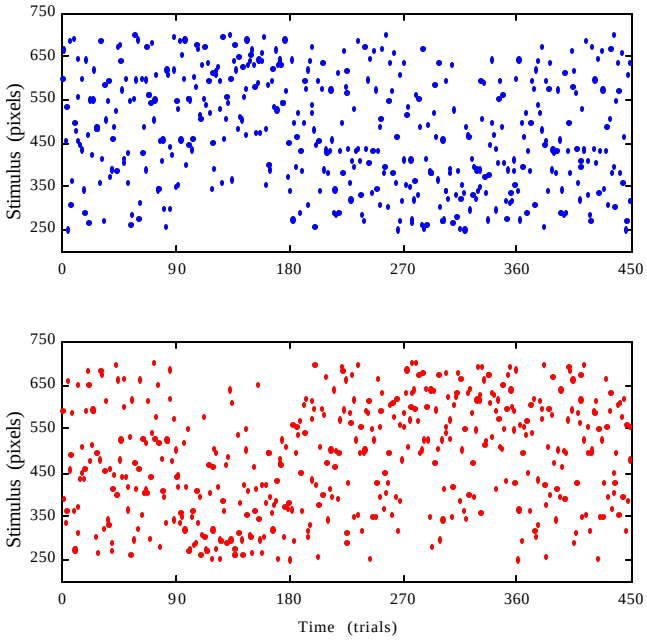
Group 2



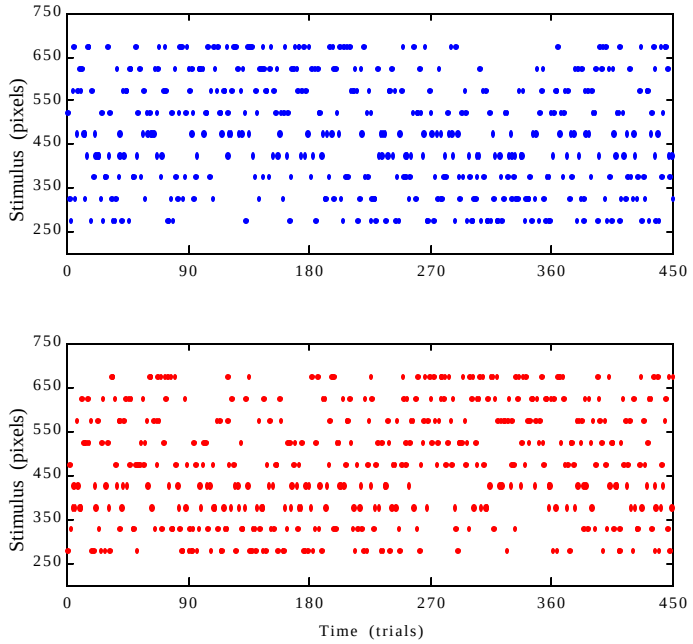


Category Rating vs Absolute Identification

- no feedback
- 450 stimulus levels



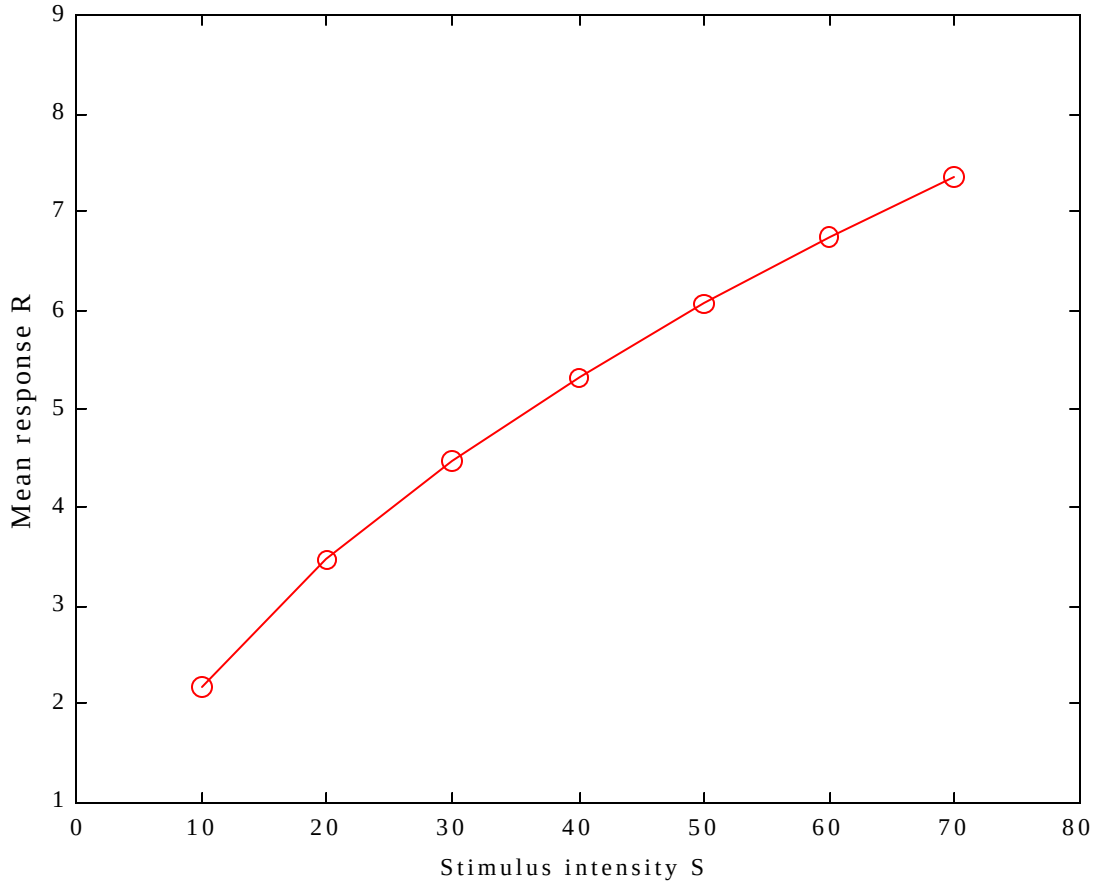
- feedback on each trial
- 9 stimulus levels only



Experiments: Details

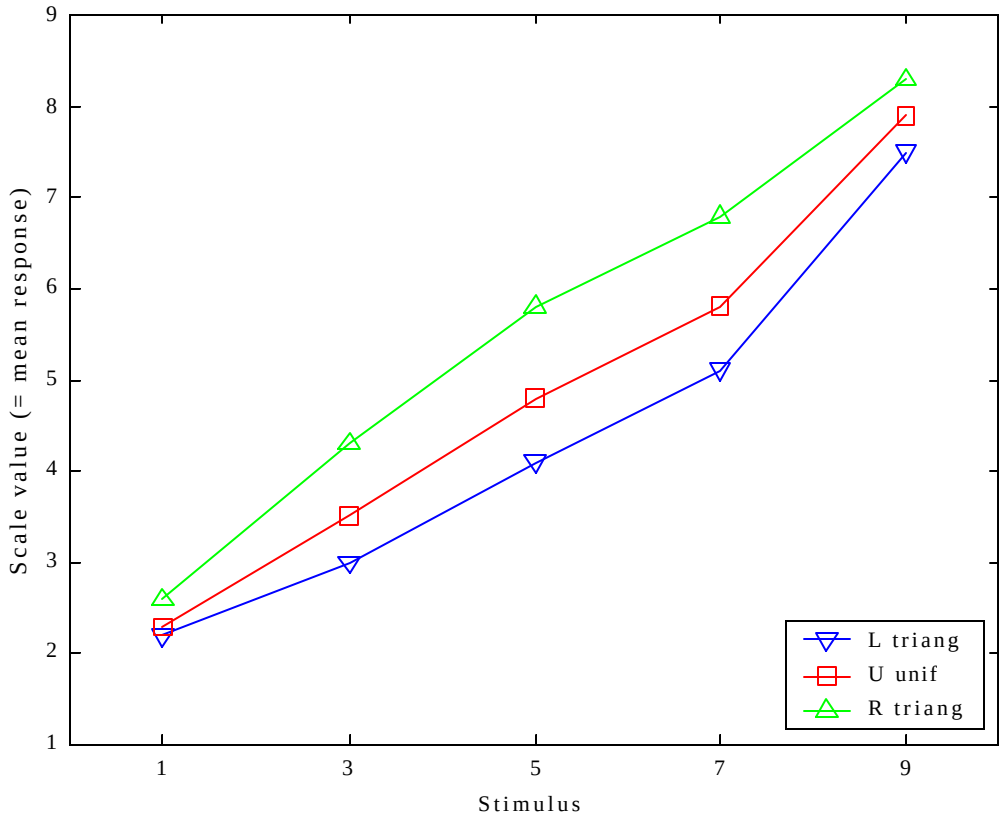
- distances b/n 250 and 700 pixels
- randomized absolute position
- 450 trials
- 17 demo trials with feedback
- 40 category-rating participants
- 24 absolute-identification participants
- 4 sec per trial, 30 min total

Stevens' Power Law: $R = aS^n$

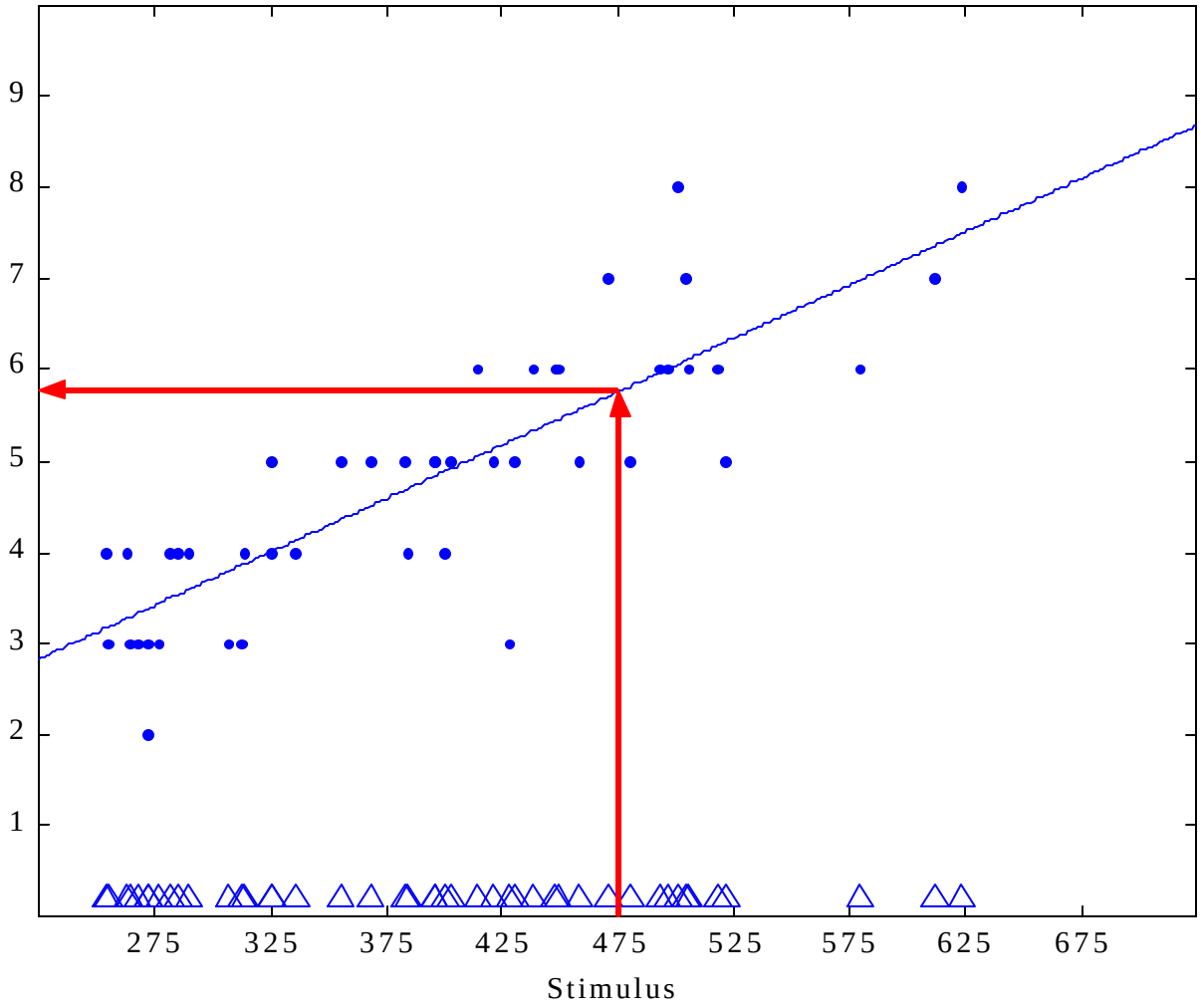


Context Effects

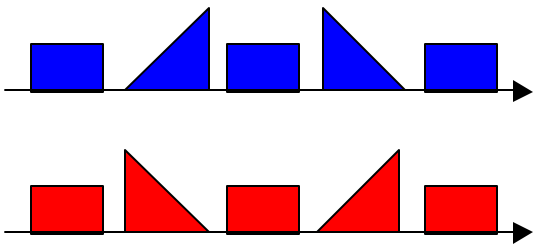
Synthetic data modeled after real data by Parducci et al. (1976)

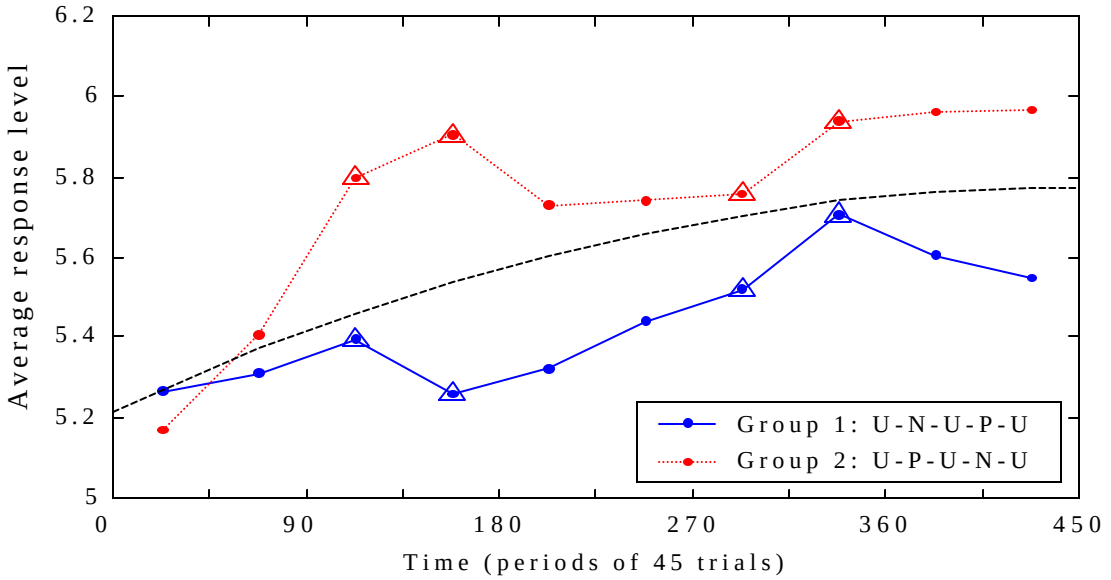


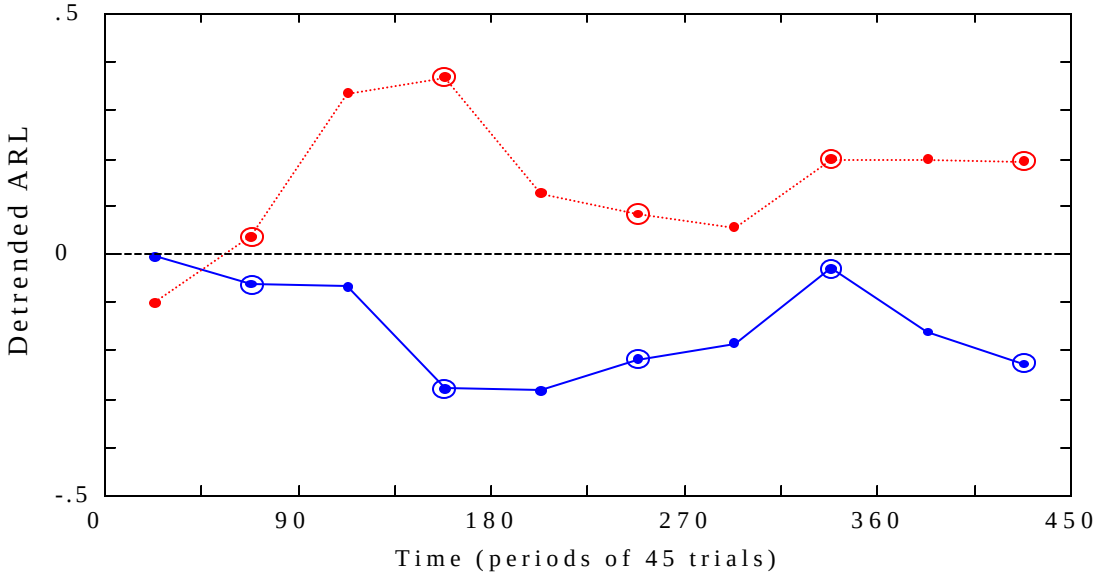
Average Response Level



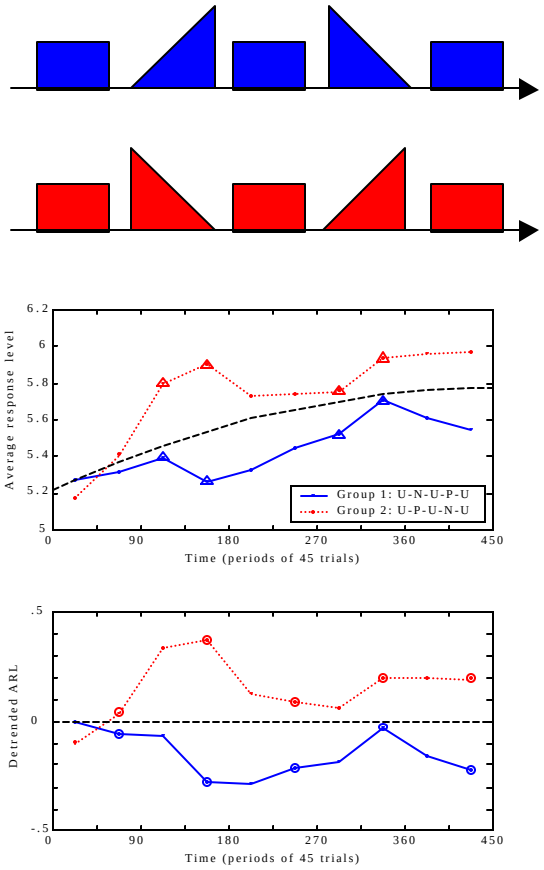
Context Effects in Category Rating







Reverse Context Effects in Absolute Identification



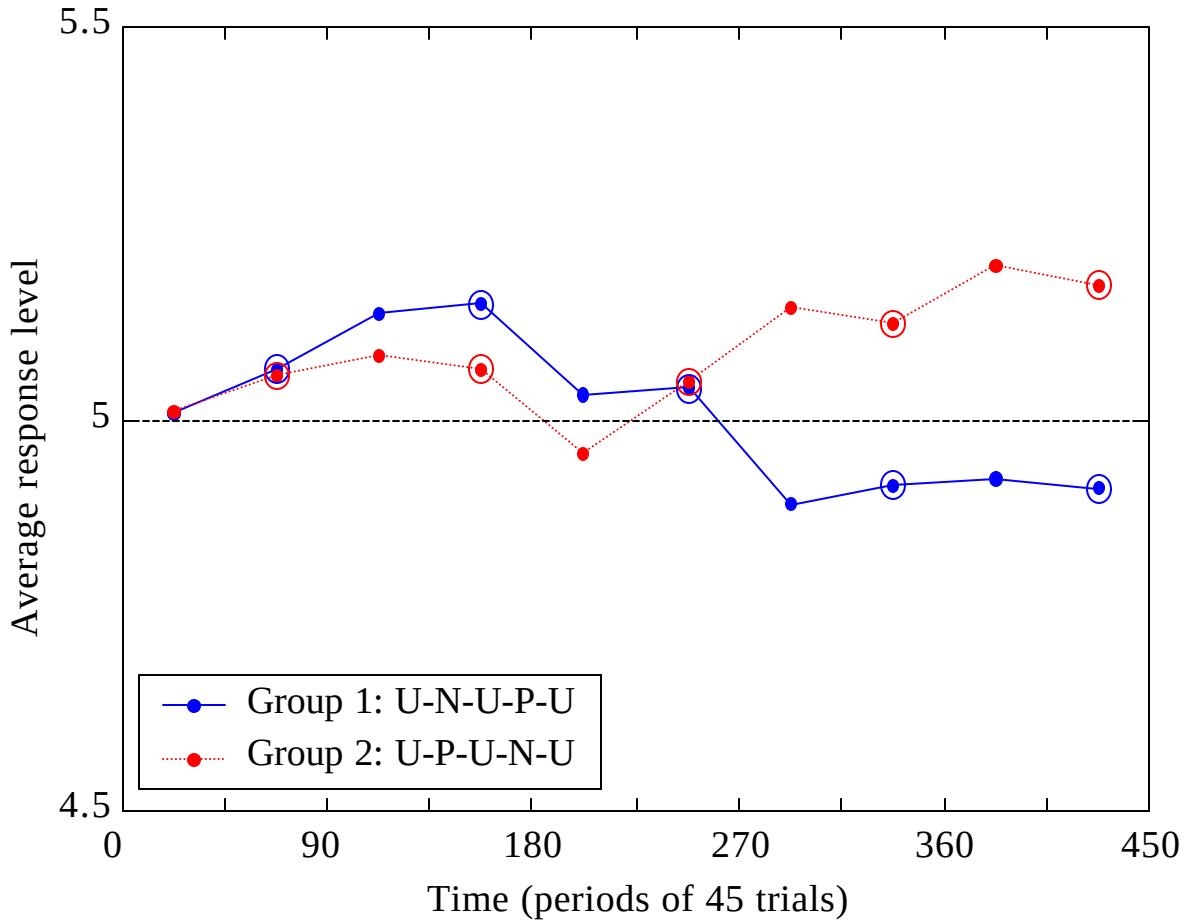
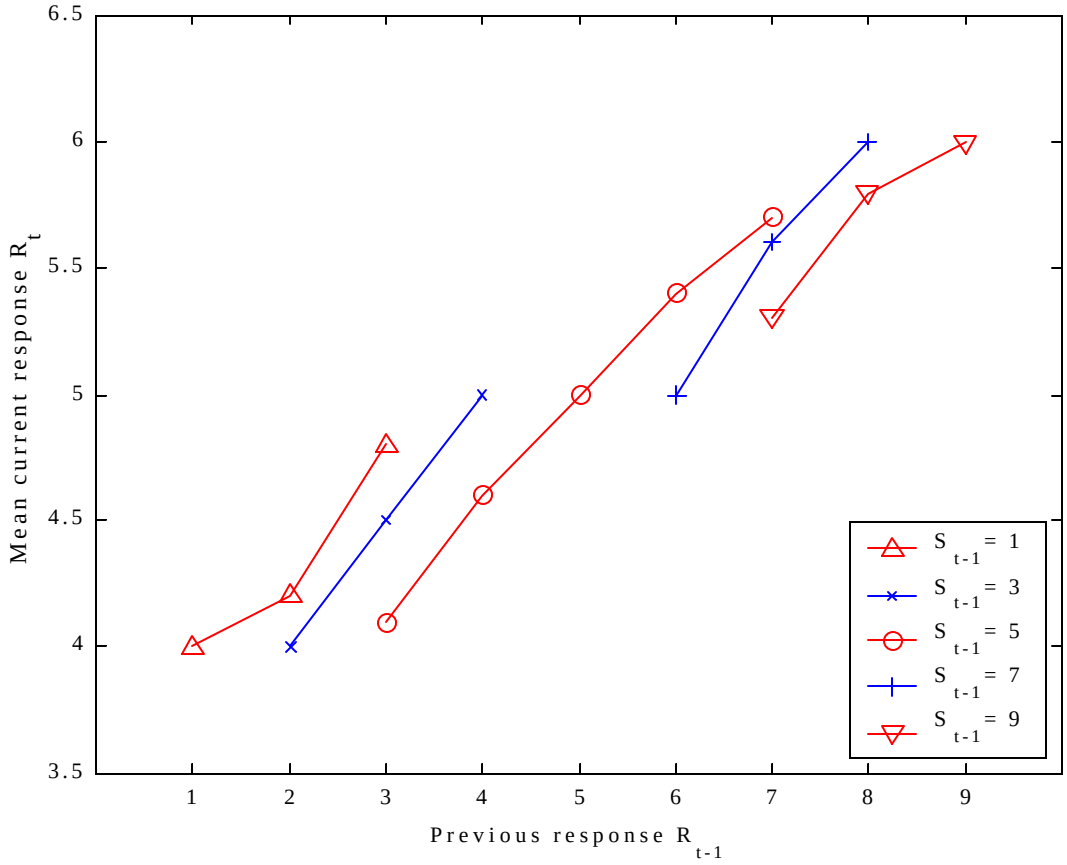


Illustration of Seqential Effects

Synthetic data modeled after real data by Petzold (1981)



Autoregression Analyses

$$R_t = const + a.S_t + b.S_{t-1} + c.R_{t-1} + err$$

- contrast: $\boldsymbol{b} < 0$
- assimilation: $\boldsymbol{c} > 0$
- interaction terms can be added

Anchors

- magnitude-label associations
- serve as prototypes
- availability

Label	Magn.	Avail.
“1”	0.103	0.567
“2”	0.214	1.208
“3”	0.297	2.091
“4”	0.402	1.445
“5”	0.521	0.382

ACT-R Interpretation

- each anchor is a *chunk*
- retrieval via *partial matching*
- *base-level activation* determines availability

$$G_i = -\left|M - A_i\right| + HB_i$$

$$P_i = \frac{\exp(G_i / T)}{\sum_j \exp(G_j / T)}$$

Connectionist Interpretation

- *pattern completion* in an attractor network

OR

- *winner-takes-all* cluster

OR

- Kohonen network

$$G_i = -\left|M - A_i\right| + HB_i$$

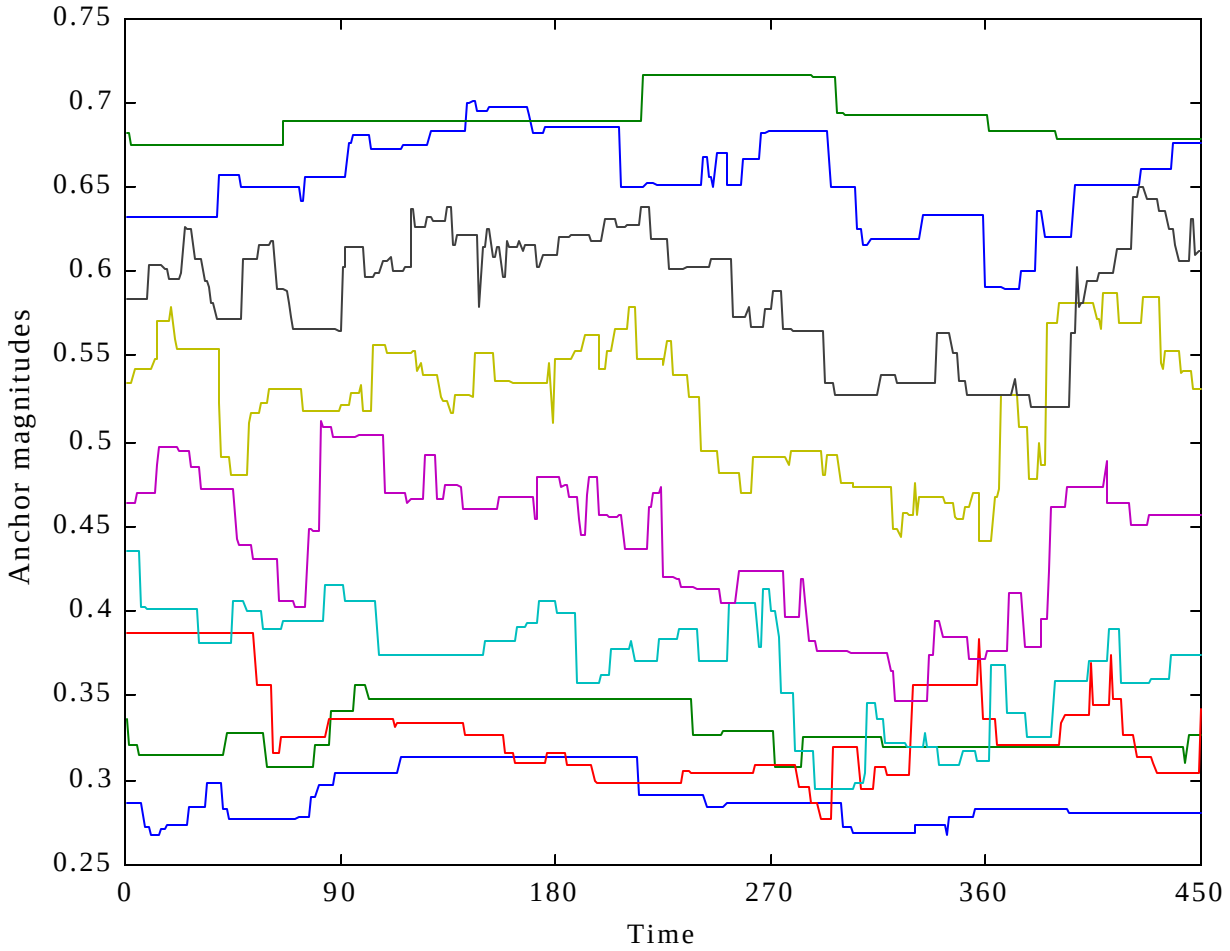
$$P_i = \frac{\exp(G_i / T)}{\sum_j \exp(G_j / T)}$$

Relation to Psychophysical “Laws”

$$M = aS\left(1 + k_p e_p\right)$$

- Stevens’ law: $\boldsymbol{M = aS^n}$ (n=1.0)
- Weber’s law: $\boldsymbol{dS/S = const}$ (k_p=0.04)
- Ekman’s law: $\boldsymbol{dM/M = const}$

Dynamics of Anchor Locations



Sequential Effects

- assimilation towards previous response R_{t-1}
- contrast with previous stimulus S_{t-1}
- interaction between the two
- intermediate-term sequential effects

$$G_i = -\left|M - A_i\right| + HB_i$$

ANCHOR vs Criterion-Setting Theory

- internal magnitude M
- anchor $<M,R>$
- recency term in BLA
- strength term in BLA
- base-level learning
- decay of base-level act.
- anchor-location learning
- exp-weighted averaging
- correction mechanism
- *<no equivalent>*
- link to memory tasks

- central effect S_{it}
- criterion $<S_c,R>$
- response indicator traces T_r
- *<no equivalent>*
- tracking mechanism
- decay of T_r (response IT)
- stabilization mechanism
- decay of T_s (stimulus IT)
- lateral shift function
- reference z_0 for each criterion
- link to 2AFC tasks