



# Bayesian Method for Repeated Threshold Estimation

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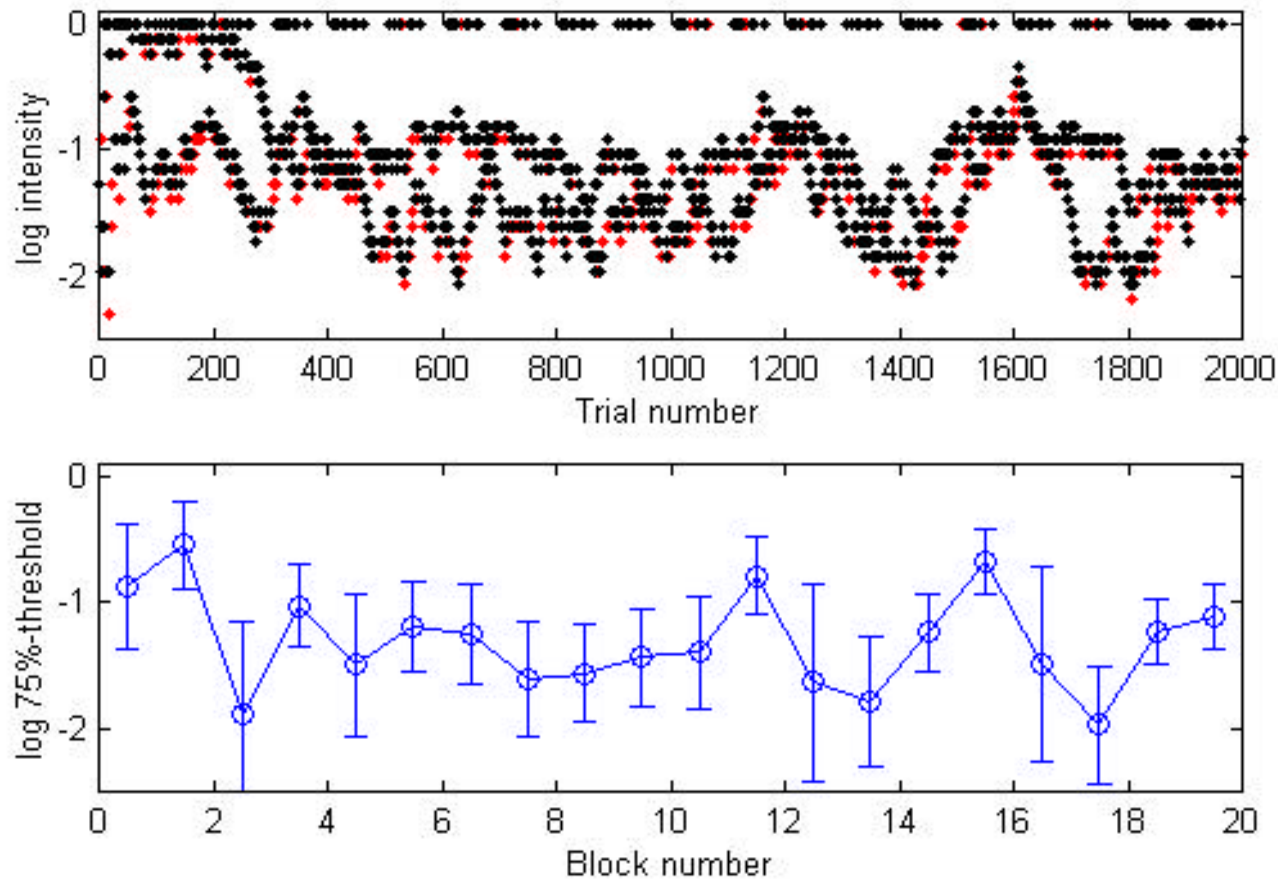
# Motivation: Perceptual Learning

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- Non-stationary thresholds
- Dynamics of learning is important
- Must use naïve observers
- Low motivation → high lapsing rates
- Slow learning → many sessions
  
- Large volume of low-quality binary data



# Objective: Data Reduction





# Isn't This a Solved Problem?

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- Up/down (Levitt, 1970)
- PEST (Taylor & Creelman, 1967)
- BEST PEST (Pentland, 1980)
- QUEST (Watson & Pelli, 1979)
- ML-Test (Harvey, 1986)
- Ideal (Pelli, 1987)
- YAAP (Tretwein, 1989)
- and many others...





# We Solve a Different Problem

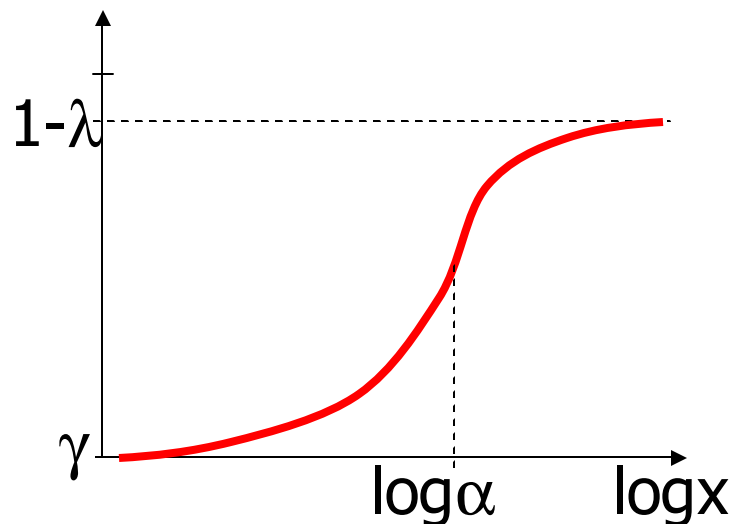
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- Standard methods:
  - Adaptive stimulus placement
  - Stopping criterion
  - Threshold estimation
- Our method:
  - Threshold estimation
  - **Integrate information across blocks**

# Weibull Psychometric Function

$$W(x; \mathbf{a}, \mathbf{b}) = 1 - \exp(-\exp((\log x - \log \mathbf{a}) \mathbf{b}))$$

$$P(x; \mathbf{a}, \mathbf{b}, \mathbf{g}, \mathbf{l}) = \mathbf{g} + (1 - \mathbf{g} - \mathbf{l})W(x; \mathbf{a}, \mathbf{b})$$



- Threshold  $\log \alpha$
- Slope  $\beta$
- Guessing rate  $\gamma$
- Lapsing rate  $\lambda$



# Two Kinds of Parameters

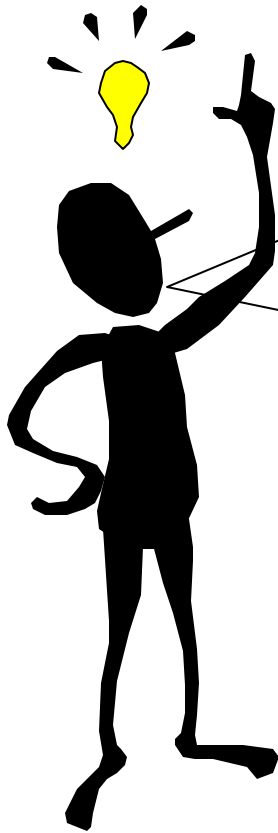
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- Threshold  $\log \alpha$
  - Slope  $\beta$
  - Guessing rate  $\gamma$
  - Lapsing rate  $\lambda$
- Parameters of interest  $\theta$
- Nuisance parameters  $\phi$

The nuisance parameters are harder to estimate but change more slowly than the threshold parameter.



# Get the Best of Both Worlds



Use long data sequences to constrain the nuisance parameters; use short sequences to estimate the thresholds.



# Joint Posterior of $\theta_k, \phi$

$$p(\mathbf{q}_k, \mathbf{f} | \mathbf{y}_k; \mathbf{y}_1 \cdots \mathbf{y}_{k-1}, \mathbf{y}_{k+1} \cdots \mathbf{y}_n) =$$
$$\underbrace{p(\mathbf{y}_k | \mathbf{q}_k, \mathbf{f})}_{\text{Likelihood of current data}} \underbrace{p(\mathbf{q}_k) p(\mathbf{f})}_{\text{Priors}} \underbrace{\prod_{i \neq k} \int p(\mathbf{q}_i) p(\mathbf{y}_i | \mathbf{q}_i, \mathbf{f}) d\mathbf{q}_i}_{\text{Information about } \phi \text{ extracted from the other data sets}}$$

Modified prior for the current block



# Two-Pass Algorithm

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- Pass 1: for each block  $i$ , calculate

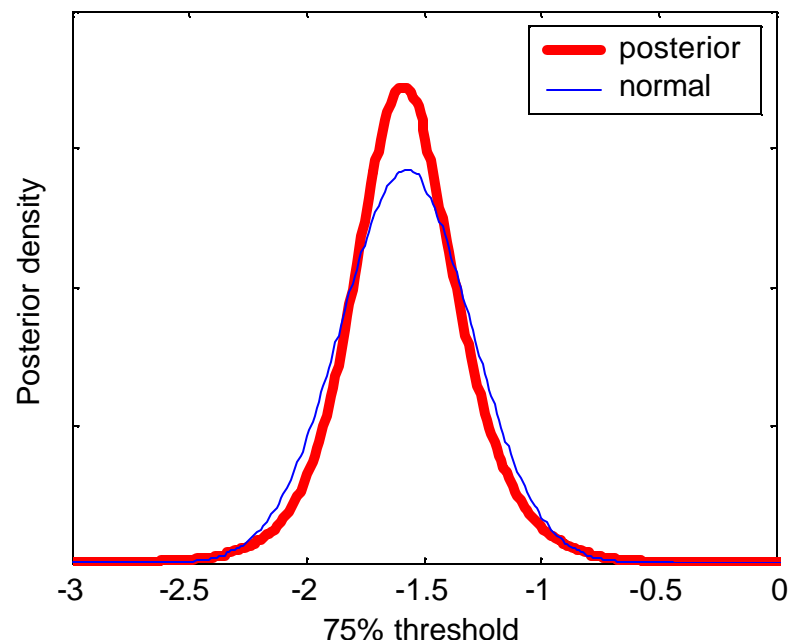
$$p(\mathbf{f} | \mathbf{y}_i) = \int p(\mathbf{q}) p(\mathbf{y}_i | \mathbf{q}, \mathbf{f}) d\mathbf{q}$$

- Pass 2: for each block  $k$ , calculate

$$p(\mathbf{q}_k, \mathbf{f} | \mathbf{y}_k) = p(\mathbf{y}_k | \mathbf{q}_k, \mathbf{f}) p(\mathbf{q}_k) p(\mathbf{f}) \prod_{i \neq k} \int p(\mathbf{f} | \mathbf{y}_i)$$

# Posterior Thresholds

$$p(T_k) = \int P^{-1}(.75; \mathbf{q}_k, \mathbf{f}) p(\mathbf{q}_k, \mathbf{f} | \mathbf{y}_k) d\mathbf{q}_k d\mathbf{f}$$





# Some Details

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- Vaguely informative priors:

$$p(\log \mathbf{a}) \propto \mathbf{N}(\mathbf{m}_a, \mathbf{S}_a)$$

$$p(\mathbf{b}) \propto \mathbf{N}(\mathbf{m}_b, \mathbf{S}_b)$$

$$p(\mathbf{I}) \propto \text{Beta}(a_1, b_1)$$

- Implemented on a grid:  $\log \alpha \times \beta \times \lambda$
- Assume  $\gamma = .5$  for 2AFC data
- MATLAB software available at <http://www.socsci.uci.edu/~apetrov/>



# Simulation 1: Stationary

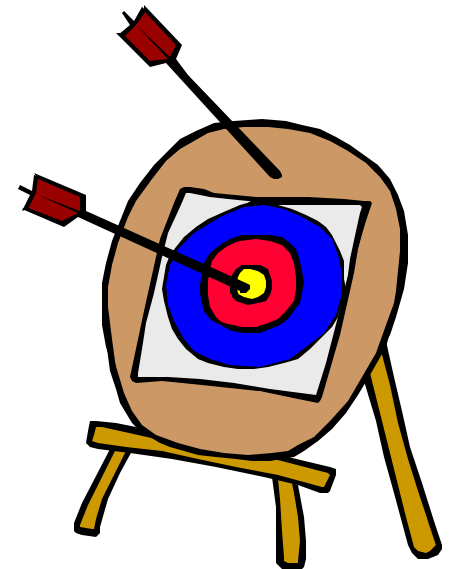
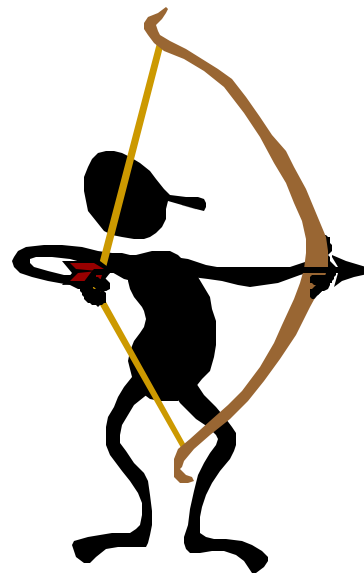
$$\log \mathbf{a} = -1.204 = \textit{const}$$

$$\mathbf{b} = 1.5$$

$$\mathbf{l} = .10$$

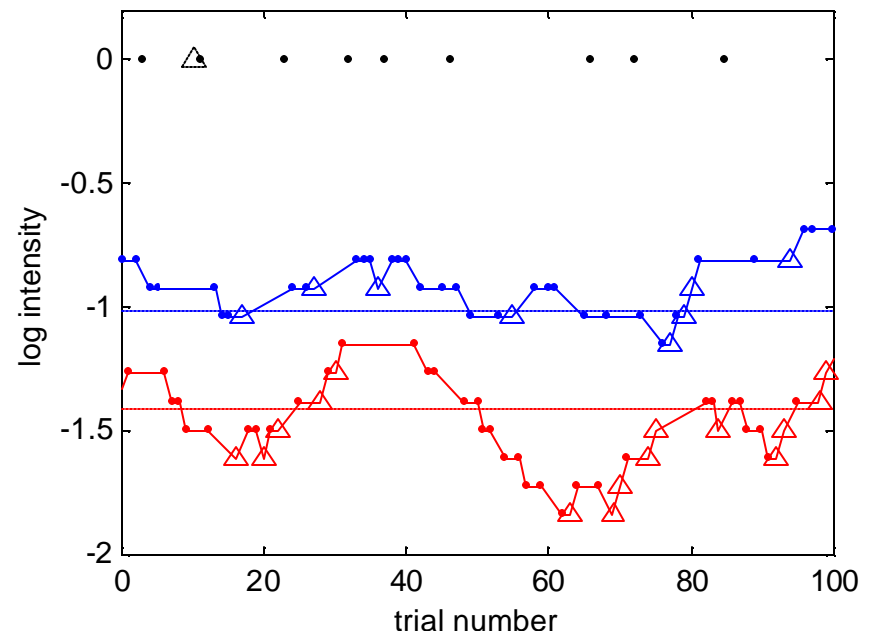


$$T_{75} = -1.217$$



# Stimulus Placement

- 2 interleaved staircases
- 100 trials/block
  - 10 catch
  - 40 x 3down/1up
  - 50 x 2down/1up
- 100 runs of 12 blocks each

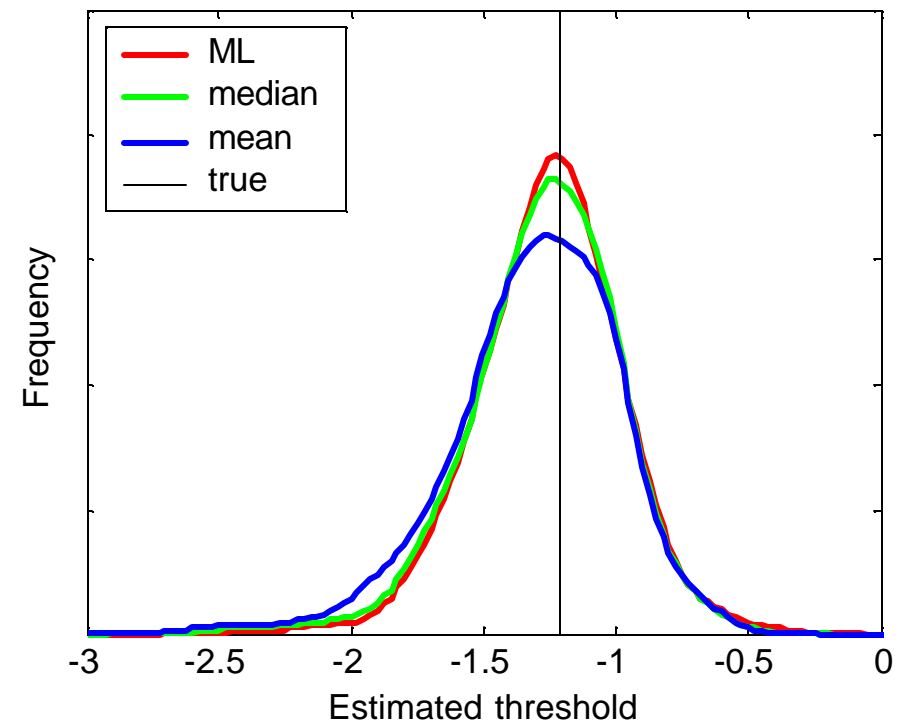


# Threshold Estimators

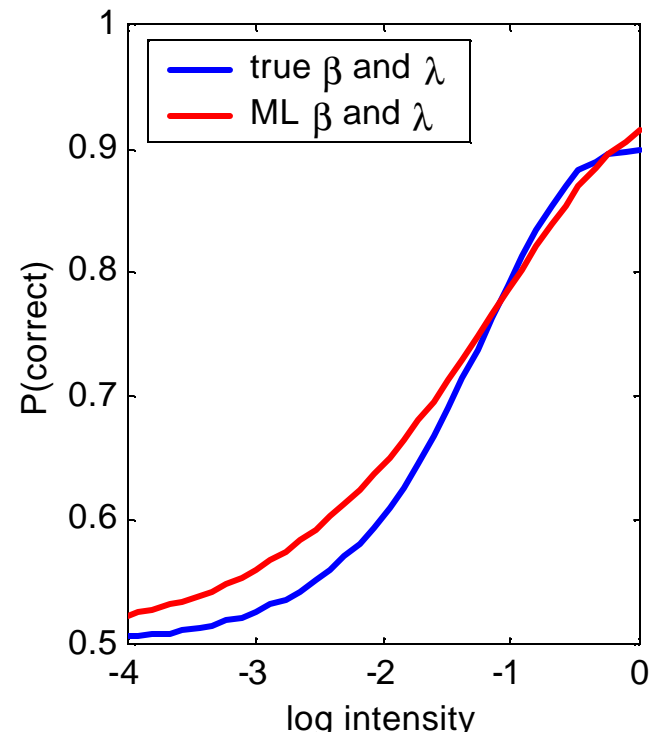
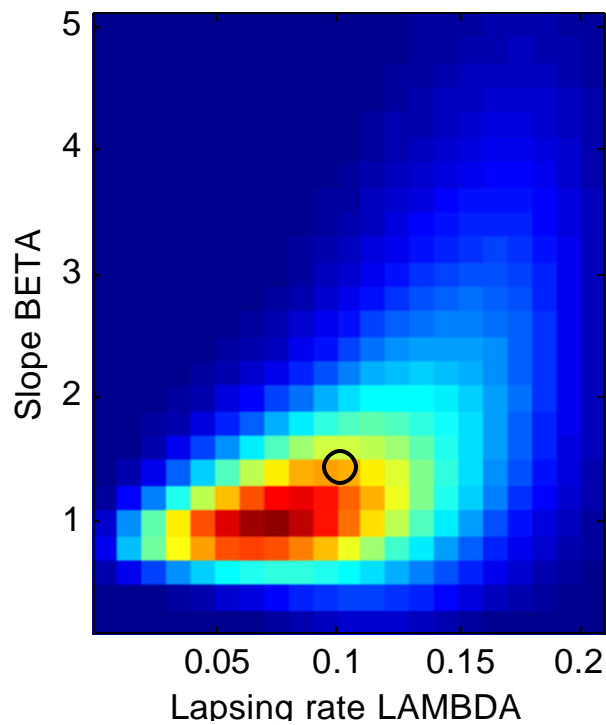
Estimator	Mean	Med	Std
ML	-1.24	-1.23	.27
Median	-1.26	-1.23	.28
Mean	-1.30	-1.27	.31
Std. dev.	0.41	0.36	.15

1200 Monte Carlo estimates

True 75% threshold = -1.217



# $\beta \times \lambda$ Distribution from Pass 1



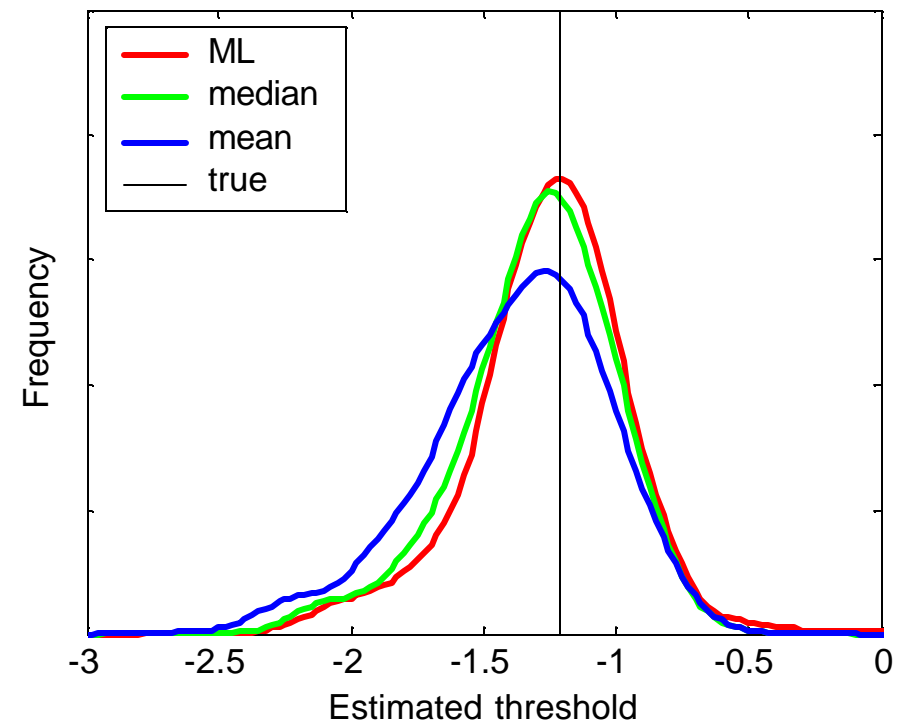
# Catch Trials Are Worthwhile

Estimator	Mean	Med	Std
ML	-1.24	-1.22	.31
Median	-1.29	-1.26	.30
Mean	-1.36	-1.33	.34
Std. dev.	0.58	0.57	.16

1200 Monte Carlo estimates

No catch trials presented

True 75% threshold = -1.217



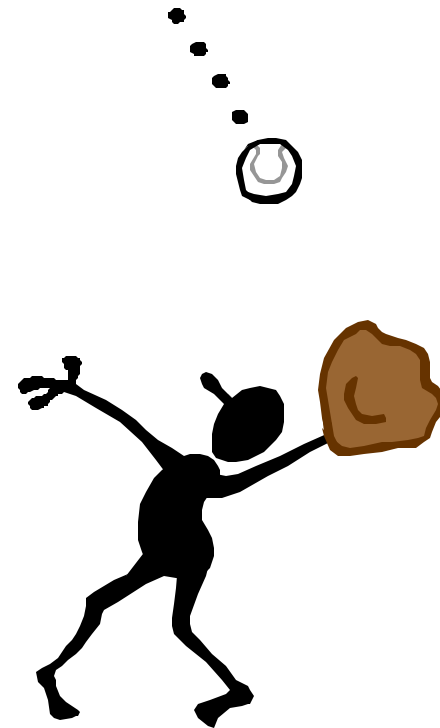
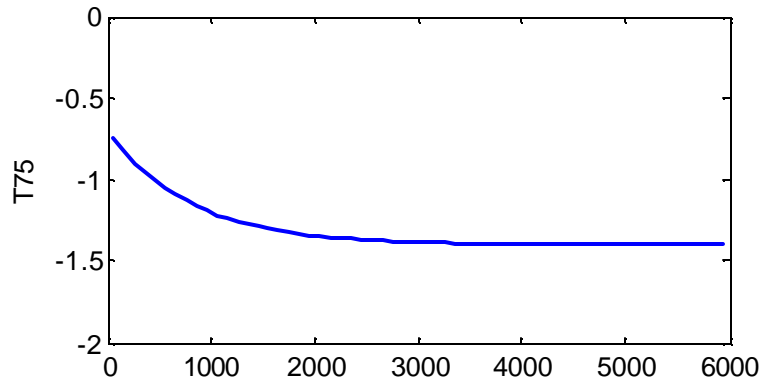


# Simulation 2: With Learning

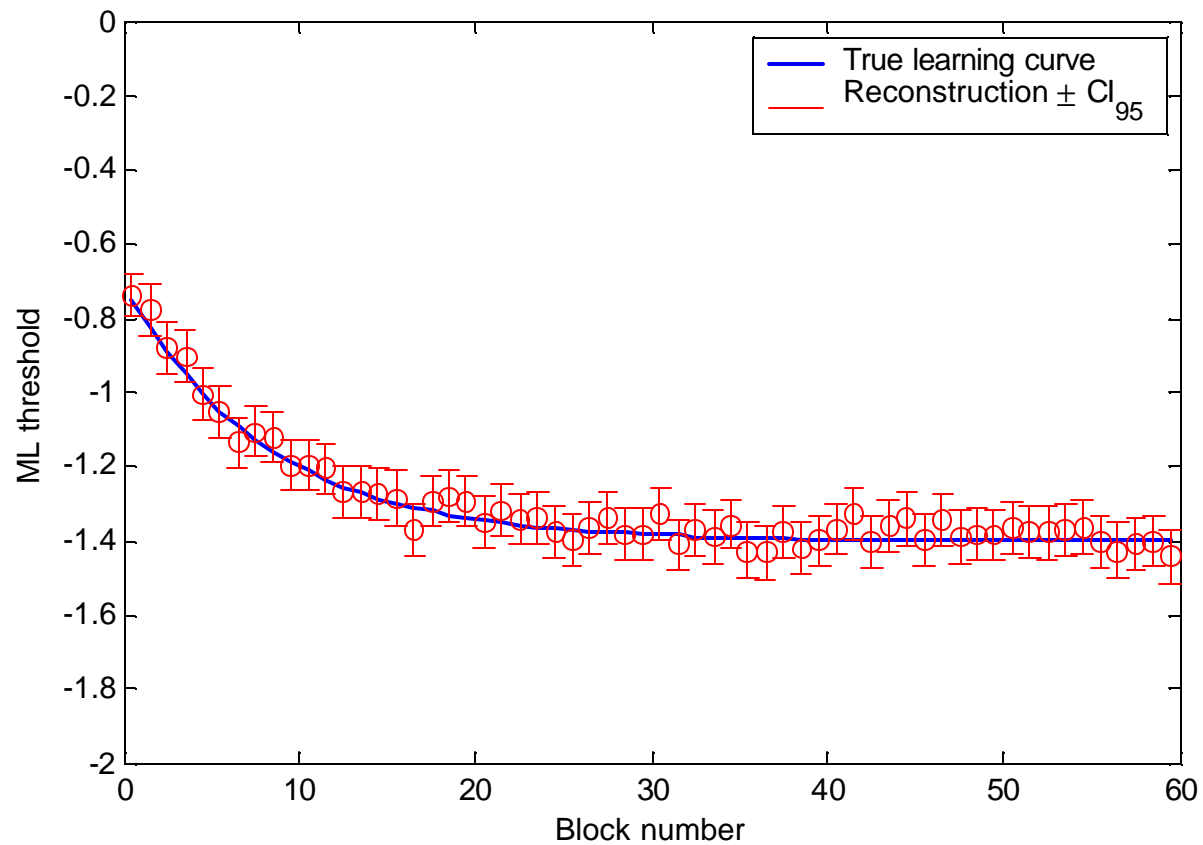
$$\log a = -0.693 (e^{-t/800} - 2)$$

$$b = 1.5$$

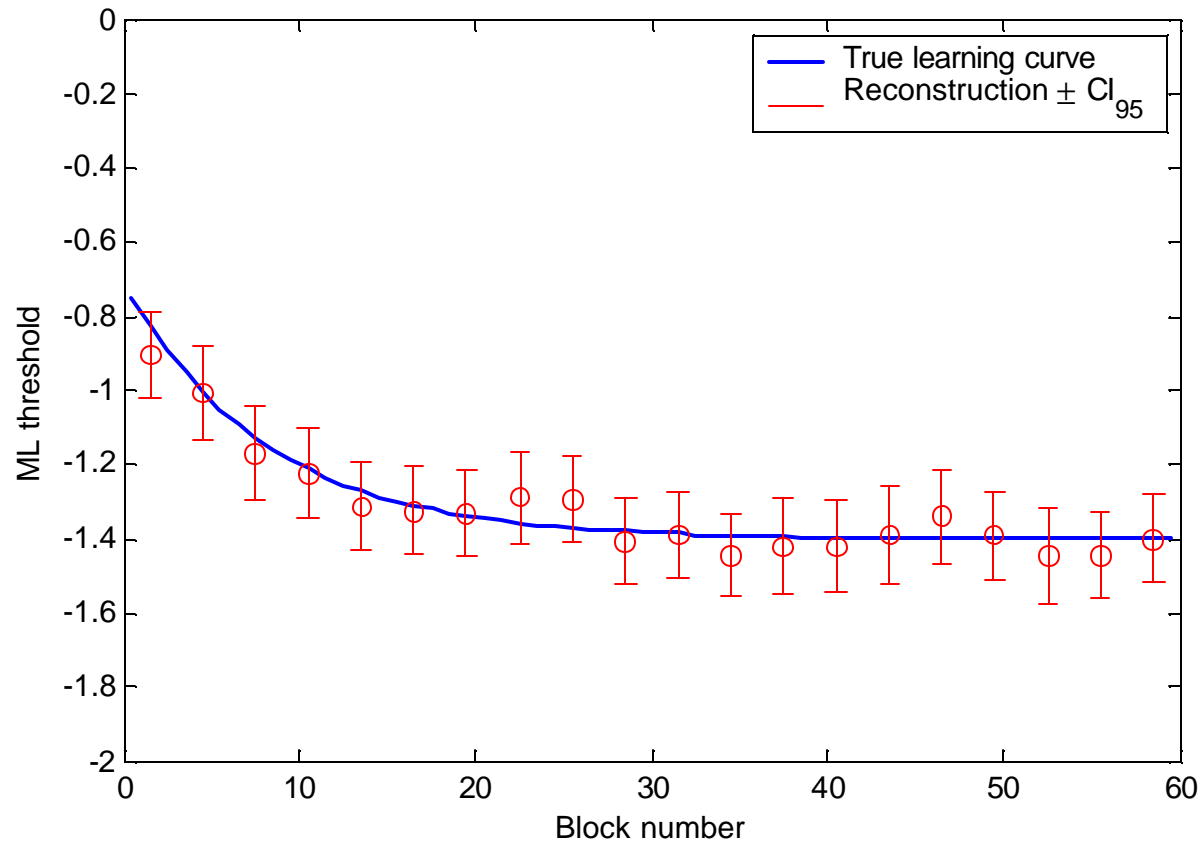
$$l = .10$$



# Group Learning Curve, N=100

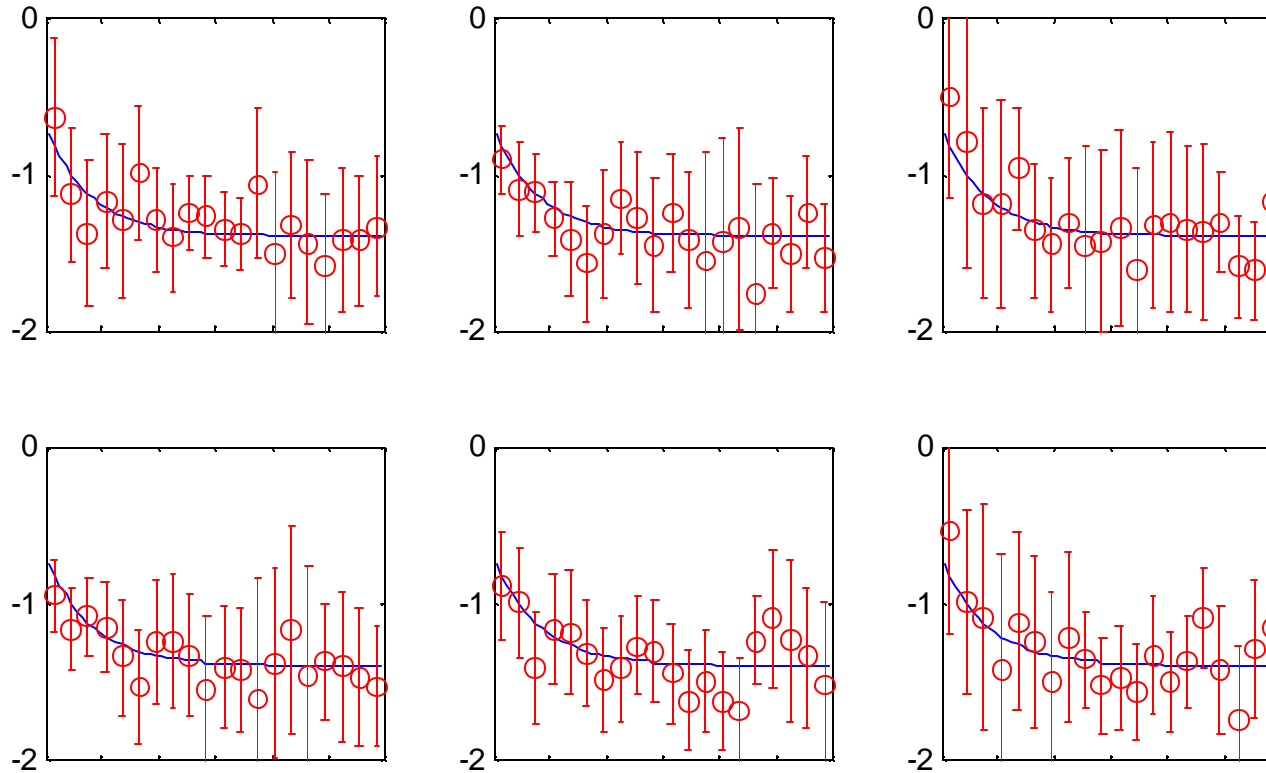


# More Realistic Sample, N=10





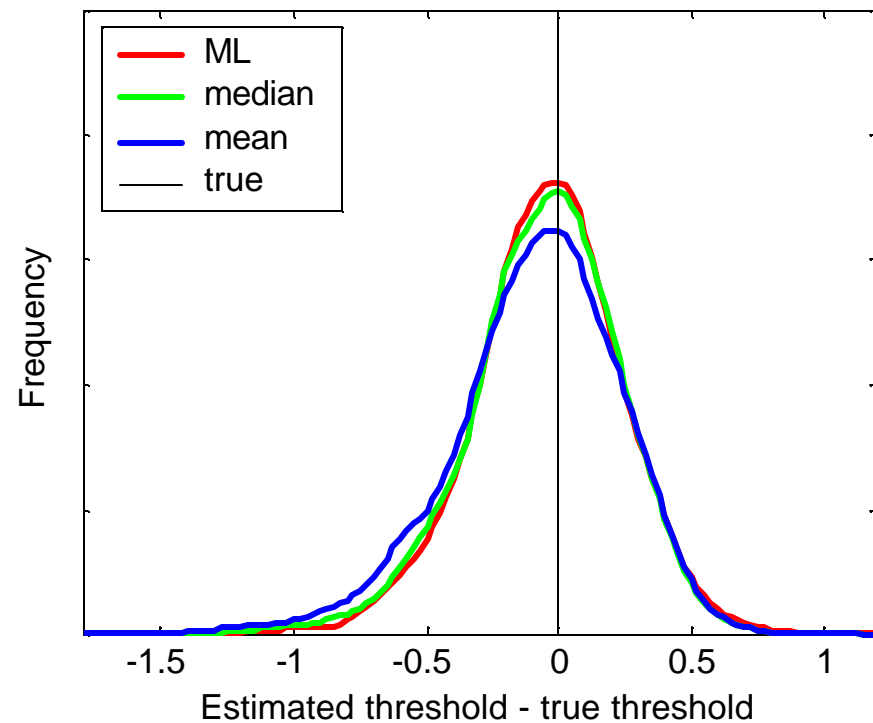
# Individual Runs



# The Method Performs Well

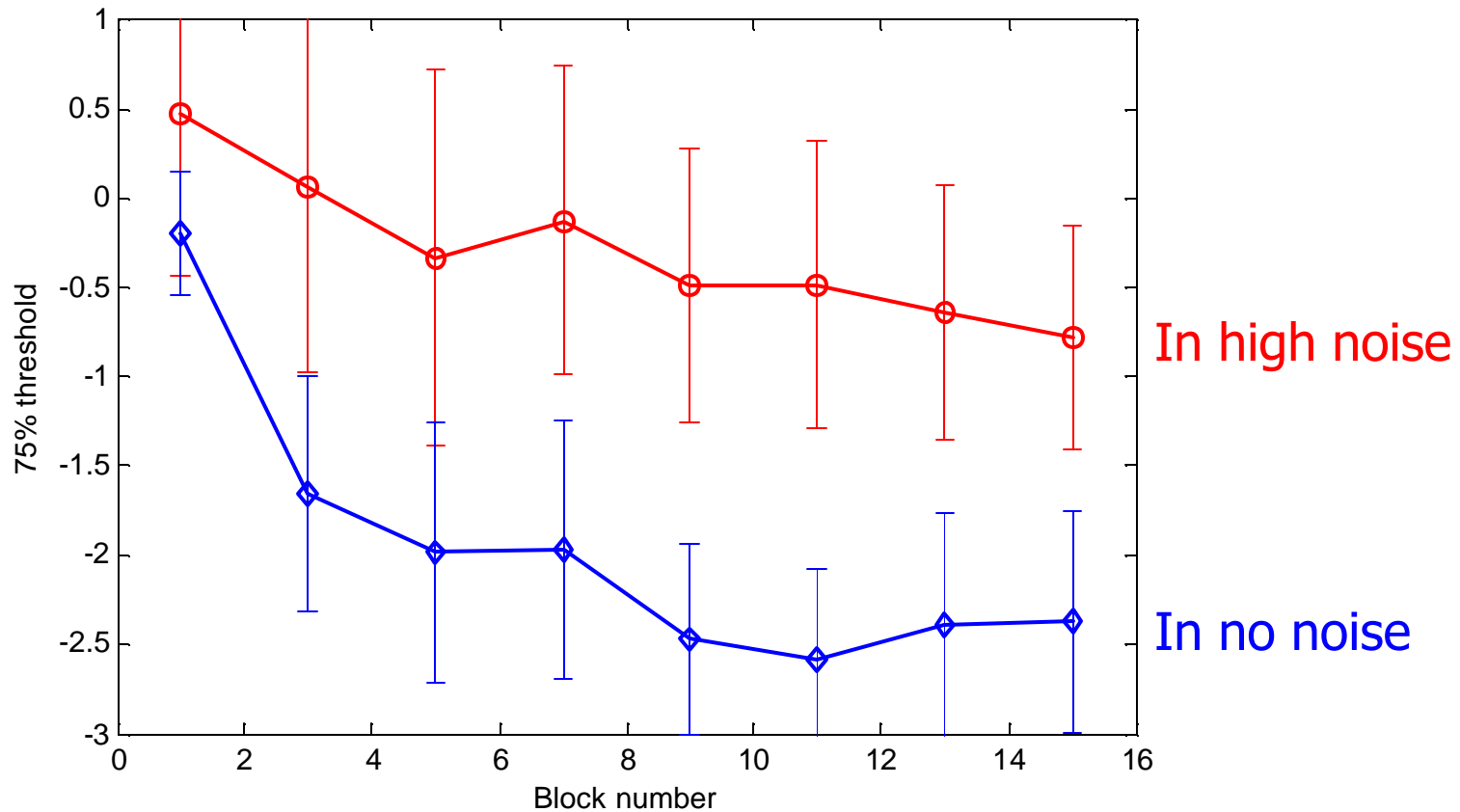
Estimator	Mean	Med	Std
ML	-0.03	-0.02	.28
Median	-0.05	-0.03	.29
Mean	-0.08	-0.05	.32
Std. dev.	0.42	0.39	.15

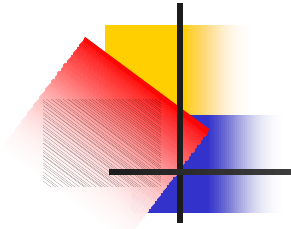
6000 Monte Carlo estimates  
Similar to the stationary case  
No systematic bias over time



# Example: Actual Data, N=8

Jeter, Doshier, Petrov, & Lu (2005)





# Future Work



- Sensitivity to priors?
- Compare with standard ML methods
- Individual differences
- Estimate slope in addition to threshold
- Non-stationary  $\beta$  and  $\lambda$ ?
- Recommended stimulus placement?
- Hierarchical models

# The End

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